DÚ 6

$$\left(\ln\left(\operatorname{arctg}\left(\sqrt{x}\right)\right)\right)'$$

$$\operatorname{tg}(x) = \frac{\sin(x)}{\cos(x)}, \quad \operatorname{tg}'(x) = \frac{\cos^2(x)+\sin^2(x)}{\cos^2(x)} = 1 + \operatorname{tg}^2(x)$$

$$\operatorname{arctg}'(x) = \frac{1}{1+\operatorname{tg}^2(\operatorname{arctg}(x))} = \frac{1}{1+x^2}$$

$$f(x) = \begin{cases} \frac{\sin(x)}{x} & x \in \mathbb{R}\setminus\{0\} \\ 1 & x = 0 \end{cases} \qquad \left(\frac{\sin(x)}{x}\right)' = \frac{x\cos(x)-\sin(x)}{x^2}$$

pro $x=0$ • z definice

• $\lim\limits_{x\to 0} \frac{x\cos(x)-\sin(x)}{x^2}$

$$f = T_n^{f,b}(x) + o\left((x-b)^n\right) \quad x \to b$$

$$g(x) \in o(x^2) \quad x\to 0 \quad \equiv \lim_{x\to 0}\frac{g(x)}{x^2} = 0$$

$$h(x) \in o(x^2) \quad x\to 0 \quad \equiv \lim_{x\to 0}\frac{h(x)}{x^2} = 0$$

aritmetika limit $\Rightarrow$ $g(x)h(x) \in o(x^4)$ $x\to 0$

$\Rightarrow \in o(x^2)$

2 policajti

$$\lim_{x\to 0}\frac{g(x)h(x)}{x^4} = 0 \quad \text{pro } x\in(-1,1)\setminus\{0\} \quad 0 \le \frac{g(x)h(x)}{x^2} \le \frac{g(x)h(x)}{x^4} \to 0$$

$$g(x) \in o(x^2) \quad x\to 0$$

$$\frac{g(x)}{x} \in o(x) \quad x\to 0$$

$$\left(p_1(x) + \overset{f_1(x)}{o(x^2)}\right)\left(p_2(x) + \overset{g(x)}{o(x^2)}\right) = p_3 + o(x^2)$$

$$x^2 \cdot f(x) \in o(x^2)$$

Per partes

$$\int x e^x \, dx = \left\| x e^x = f(x)\,G(x) \right. \qquad x = G(x) \quad g(x) = 1$$
$$e^x = f(x) \quad F(x) = e^x$$

$$= x e^x - \int e^x \, dx = e^x(x-1) + c$$

$$\overset{\shortparallel}{e^x}$$

$\int \ln(x)\,dx$ na přednášce

$f(x) = 1 \quad F(x) = x$

$G(x) = \ln(x) \quad g(x) = \frac{1}{x}$

$$\int \cos^2(x)\,dx \qquad = \sin(x)\cos(x) + \int \sin^2(x)\,dx =$$

$f(x) = \cos(x)$, $F(x) = \sin(x)$

$G(x) = \cos(x)$, $g(x) = -\sin(x)$

~~$f(x) = \sin(x)$ $F(x) = -\cos(x)$~~

~~$G(x) = \sin(x)$~~ $g(x) = \cos(x)$

$$= \sin(x)\cos(x) + \int 1-\cos^2(x)\,dx = \sin(x)\cos(x) + x - \int \cos^2(x)\,dx$$

$$\cancel{2}\int \cos^2(x)\,dx = \frac{\sin(x)\cos(x)+x}{2}$$

$$\int e^x \sin(e^x)\,dx = \int \sin(s)\,ds = -\cos(s) + c = -\cos(e^x) + c$$

$s = e^x$

$$\frac{ds}{dx} = e^x \quad \Rightarrow ds = e^x\,dx$$

$$\int \frac{x}{x^2+1}\,dx = \frac{1}{2}\int\frac{ds}{s} = \frac{1}{2}\ln(|s|) + c = \frac{1}{2}\ln(x^2+1) + c$$

$s = x^2+1$

$ds = 2x\,dx$

$$\int \frac{2x+3}{x^2-2x+5}\,dx = \int \frac{2x-2 \; +5}{x^2-2x+5}\,dx = \int \frac{2x-2}{x^2-2x+5}\,dx + 5\int \frac{1}{x^2-2x+5}\,dx =$$

$s = x^2-2x+5$

$ds = 2x-2\,dx$

$$= \int \frac{ds}{s} + 5\int\frac{1}{x^2-2x+5}\,dx$$

$\ln(x^2-2x+5)$

$$\int \frac{1}{x^2-2x+5}\,dx = \int \frac{1}{x^2-2x+1-1+5}\,dx = \int \frac{1}{4+(x-1)^2}\,dx = \frac{1}{4}\int \frac{1}{1+\frac{(x-1)^2}{4}}\,dx =$$

$$= \frac{1}{4}\int \frac{1}{1+\left(\frac{x-1}{2}\right)^2}\,dx = \frac{1}{2}\int \frac{1/2}{1+\left(\frac{x-1}{2}\right)^2}\,dx = \frac{1}{2}\int \frac{1}{1+s^2}\,ds =$$

$s = \frac{x-1}{2} \quad ds = \frac{1}{2}\,dx$

$$= \frac{1}{2}\operatorname{arctg}(s) = \frac{1}{2}\operatorname{arctg}\left(\frac{x-1}{2}\right)$$

$$\int \frac{x^2}{\sqrt{1-x^2}}\,dx = \int \frac{\sin^2(s)}{\sqrt{1-\sin^2(s)}}\cancel{\cos(s)}\,ds = \int \sin^2(s)\,ds =$$

$x = \sin(s)$ $\quad$ $\overset{\shortparallel}{\cancel{\cos}(s)}$

$dx = \cos(s)\,ds$

$$= \frac{1}{2}\left(s + \cos(s)\sin(s)\right) =$$

$D_0 \ (-1,1)$

$\sin(x)$ na $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ $= \frac{1}{2}\left(\arcsin(x) + x\cos\left(\arcsin(x)\right)\right)$

$\sin\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) = (-1,1)$

$\sin'(x) = \cos(x)$ nenabývá 0 na $(-1,1)$