

Úlohy ke cvičení

Věta 1 (Binomická věta). $(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^{n-k} y^k$.

Úloha 1. Dokažte kombinatorickou úvahou:

$$1. \binom{n-1}{k-1} + \binom{n-1}{k} = \binom{n}{k}$$

$$2. \sum_{k=0}^n \binom{n}{k} = 2^n$$

$$3. \sum_{k=0}^n (-1)^k \binom{n}{k} = 0$$

$$4. \sum_{k=0}^r \binom{n}{k} \binom{m}{r-k} = \binom{m+n}{r}$$

$$5. \sum_{k=r}^n \binom{k}{r} = \binom{n+1}{r+1}$$

$$6. \sum_{k=0}^n k \binom{n}{k} = n2^{n-1}$$