# AGT 6th tutorial*

30 November 2023

## 1 Regret minimization and CCE

For a game $G = (P, A, C)$ in a normal game for $n$ players the probability distribution $p(a)$ on the set $A$ is a *correlated equilibrium* in $G$, if $\sum_{a_{-i} \in A_{-i}} C_i(a_i; a_{-i}) p(a_i; a_{-i}) \leq \sum_{a_{-i} \in A_{-i}} C_i(a'_i; a_{-i}) p(a_i; a_{-i})$ for all players $i \in P$ and all $a_i, a'_i \in A_i$. Probability distribution $p(a)$ on the set $A$ is a *coarse correlated equilibrium* in $G$, if $\sum_{a \in A} C_i(a) p(a) \leq \sum_{a \in A} C_i(a'_i; a_{-i}) p(a)$ for all players $i \in P$ and all $a'_i \in A_i$.

**Exercise 1.** *Formally prove that every correlated equilibrium is a coarse correlated equilibrium.*

**Exercise 2.** *Compute all CCE from the prisoners dilemma.*

| | T | S |
|---|---|---|
| T | (2,2) | (0,3) |
| S | (3,0) | (1,1) |

Table 1: Game from exercise 2

There are $N$ available actions $X = \{1, \ldots, N\}$ and at each time step $t$ the online algorithm $A$ selects a probability distribution $p^t = (p_1^t, \ldots, p_N^t)$ over $X$. After the distribution $p^t$ is chosen at time step $t$, the adversary chooses a loss vector $\ell^t = (\ell_1^t, \ldots, \ell_N^t) \in [-1, 1]^N$, where the number $\ell_i^t$ is the loss of action $i$ in time $t$. The algorithm $A$ then experiences loss $\ell_A^t = \sum_{i=1}^N p_i^t \ell_i^t$. After $T$ steps, the loss of action $i$ is $L_i^T = \sum_{t=1}^T \ell_i^t$ and the loss of $A$ is $L_A^T = \sum_{t=1}^T \ell_A^t$. The *external regret* of $A$ is $R_A^T = \max_{i \in X} \{L_A^T - L_i^T\}$.

**Exercise 3** (*)**.** *Prove the following statements about lower bounds for external regret.*

(a) *Let $N$ a $T$ be natural numbers such that $N$ is power of 2 and $T < \log_2 N$, then there exists choice of random vectors of losses from $\{0, 1\}$ such that every online algorithm $A$ satisfy $\mathbb{E}[L_A^T] \geq T/2$ and $L_{min}^T = 0$*

(b) *For $N = 2$, there exists a choice of random vectors of losses from $\{0, 1\}$ such that every online algorithm $A$ satisfy $\mathbb{E}[L_A^T - L_{min}^T] \geq \Omega(\sqrt{T})$.*

*Informace o cvičení naleznete na http://kam.mff.cuni.cz/~ryzak/