INTRODUCTION

What are correlated or ε-Nash equilibria?

• **ε-NASH** is an approximation NE (Every NE is ε-NE)

Let $G = (P, A, u)$ be a normal-form game of n players and let $\varepsilon > 0$. A strategy profile $s = (s_1, \ldots, s_n)$ is an *ε-Nash equilibrium* if, for every player $i \in P$ and for every strategy $s'_i \in S_i$, we have $u_i(s_i; s_{-i}) \geq u_i(s'_i; s_{-i}) - \varepsilon$.

• **Correlated** eq. (CE)

→ New concept of equilibrium

→ We can think of CE as a traffic light telling each player to go for a specific action where an action $a \in A$ is draw randomly from A according to p

→ Difference from the definition of NE is that we do not need to be concerned in mixed strategies but only in pure ones

Let p be a probability distribution on A, that is, $p(a) \geq 0$ for every $a \in A$ and $\sum_{a \in A} p(a) = 1$. The distribution p is a *correlated equilibrium* in G if

$$\sum_{a_{-i} \in A_{-i}} u_i(a_i; a_{-i}) p(a_i; a_{-i}) \geq \sum_{a_{-i} \in A_{-i}} u_i(a'_i; a_{-i}) p(a_i; a_{-i})$$

for every player $i \in P$ and all pure strategies $a_i, a'_i \in A_i$.

Definition of CE says: Changing a pure strategy while being in CE can not get better utility for a player.

$\boxed{\text{EXERCISES}}$

Exercise 1. *Show that, in every normal-form game $G = (P, A, u)$, every convex combination of correlated equilibria is a correlated equilibrium.*

$\boxed{\text{HINTS}}$: i) Take p' and p both CE (p', p Probability distributions over A)

ii) take $p'' = tp + (1-t)p'$
$t \in [0,1]$

iii) use formula from definition for p''

Exercise 2. *Let $G = (P = \{1, 2\}, A, u)$ be a normal-form game of two players with $A_1 = \{U, D\}$ and $A_2 = \{L, R\}$ with payoff function u depicted in Table 1.*

	L	R
U	(1,1)	(0,0)
D	$(1 + \frac{\varepsilon}{2}, 1)$	(500,500)

Table 1: A game from Exercise 2.

Show that there is an ε-Nash equilibrium s of G such that $u_i(s') > 10u_i(s)$ for every $i \in P$ and every Nash equilibrium s' of G. In other words, there might be games where some ε-Nash equilibria are far away from any Nash equilibrium.

HINTS: i) Find all NE
(analyze and compute utilities
(or see NE from payoffs)

ii) Find pure strategy ε-NE
with property being at least
10 times lower utility
then all NE

Results 2) 1) 1 NE, $s_1 = (p_U \mid p_D) = (0, 1)$
$s_2 = (p_L \mid p_R) = (0, 1)$

2) ε-NE ... $s_1 = (1, 0), s_2 = (1, 0)$
$\Rightarrow$ action (U, L)

Exercise 3. *Let $G = (P = \{1, 2\}, A, u)$ be a normal-form game of two players with $A_1 = \{U, D\}$ and $A_2 = \{L, R\}$ with payoff function u depicted in Table 2.*

	L	R
U	(6,6)	(2,7)
D	(7,2)	(0,0)

Table 2: A game from Exercise 3.

(a) *Compute all Nash equilibria of G and draw the convex hull of Nash equilibrium payoffs.*

(b) *Is there any correlated equilibrium of G (for some ditribution p) that yields payoffs outside this convex hull?*

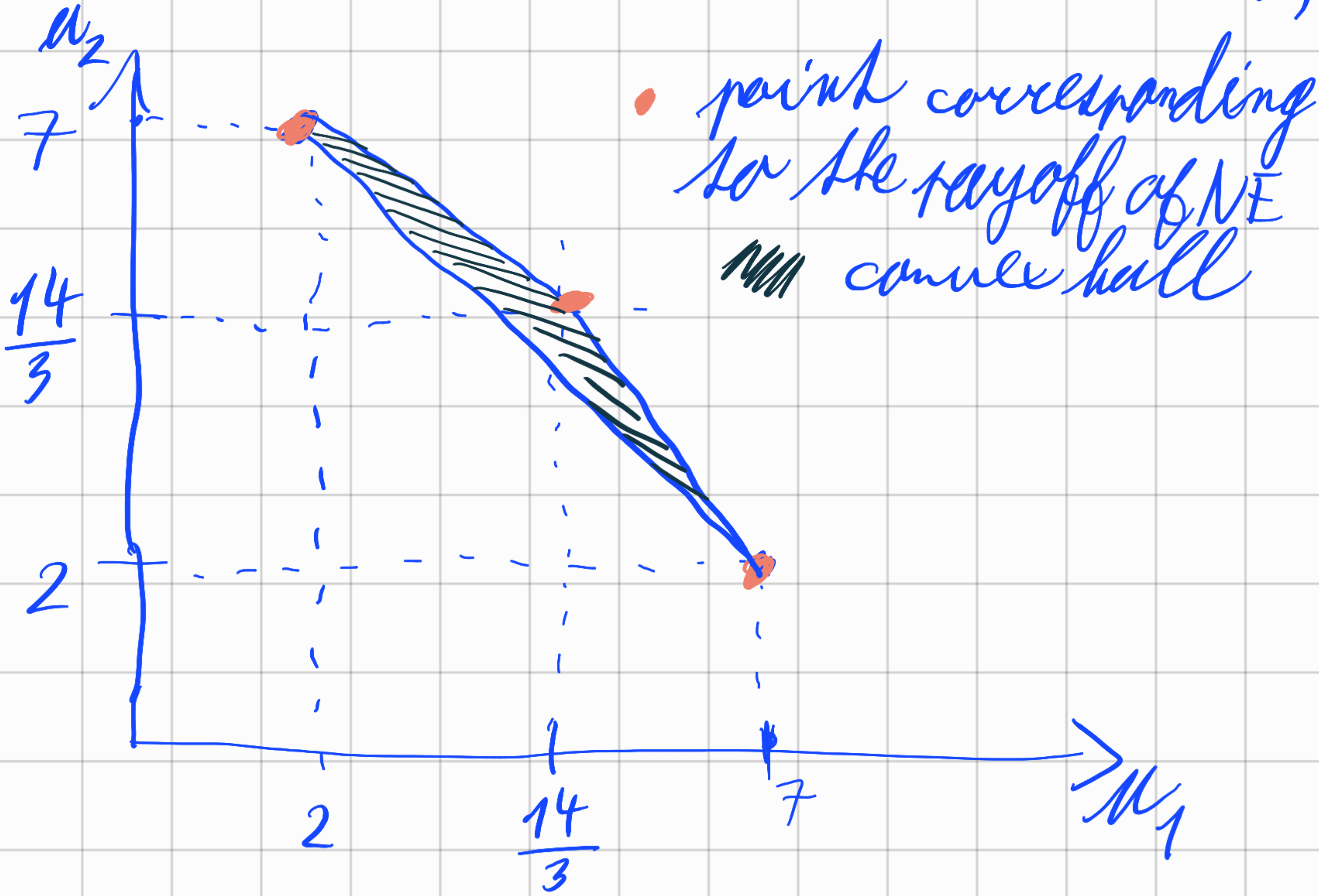
$\boxed{\text{HINTS:}}$ i) Compute all NE

ii) From analysis of NE we (analysis of utilities) can see that convex hull of payoffs given by NEs is the convex hull of all the payoffs

iii) Play strategy (D,R) with 0 probability

Results a) all NE: $(s_1, s_2) = \left(\begin{pmatrix} p_U \\ p_D \end{pmatrix}, \begin{pmatrix} p_L \\ p_R \end{pmatrix} \right)$

$$\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right), \left(\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right), \left(\begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix}, \begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \end{pmatrix} \right)$$



b) e.g.

$$p(U,R) = p(U,L) = p(D,L) = \frac{1}{3}$$

$\boxed{\text{Note.}}$ Convex hull of payoffs given by correlated equilibria is superset of the convex hull given by NEs.

C(NE) ..... convex hull of payoffs given by NE

C(CE) ... convex hull of payoffs given by CE

Exercise 4. *Spočítejte všechna korelovaná ekvilibria ve hře Vězňovo dilema.*

	T	S
T	(-2,-2)	(0,-3)
S	(-3,0)	(-1,-1)

• HINT: (i) use the definition of CE (all inequalities)

Results: CE is only $p(T,T)=1$

That is the only CE is also NE.

Note: all NE are CE but not the other way around (see exercise 3)