Algorithmic game theory – Tutorial 6*

December 21, 2023

1 Games in extensive form

The *sequence form* of an imperfect-information game G is a 4-tuple (P, S, u, C) where P is a set of n players, $S = (S_1, \dots, S_n)$, where S_i is a set of sequences of player i, $u = (u_1, \dots, u_n)$, where $u_i \colon S \to \mathbb{R}$ is the payoff function of player i, and $C = (C_1, \dots, C_n)$ is a set of linear constraints on the realization probabilities of player i.

Exercise 1. *Construct an extensive form of the Game of chicken from Table 1 and write its sequence form and the linear complementarity problem for finding Nash equilibria in this game.*

	Turn	Straight
Turn	(0,0)	(-1,1)
Straight	(1,-1)	(-10,-10)

Table 1: A normal form of the Game of chicken.

2 Mechanism design basics

An auction is *dominant-strategy incentive-compatible (DSIC)* if it satisfies the following two properties. Every bidder has a dominant strategy: *bid truthfully*, that is, set his bid b_i to his private valuation v_i. Moreover, the utility of every truth-telling bidder is guaranteed to be non-negative.

Theorem 1 (Myerson's lemma)**.** *In a single-parameter environment, the following three claims hold.*

(a) *An allocation rule is implementable if and only if it is monotone.*

(b) *If an allocation rule x is monotone, then there exists a unique payment rule p such that the mechanism (x, p) is DSIC (assuming that $b_i = 0$ implies $p_i(b) = 0$).*

(c) *The payment rule p is given by the following explicit formula*

$$p_i(b_i; b_{-i}) = \int_0^{b_i} z \cdot \frac{\mathrm{d}}{\mathrm{d}z} x_i(z; b_{-i}) \, \mathrm{d}z$$

for every $i \in \{1, \dots, n\}$.

Exercise 2. *Consider a single-item auction with at least three bidders. Prove that selling the item to the highest bidder at a price equal to the third-highest bid, yields an auction that is not dominant-strategy incentive compatible (DSIC).*

Exercise 3. *Use Myerson's Lemma to prove that the Vickrey auction is the unique single-item auction that is DSIC, always awards the good to the highest bidder, and charges the other bidders 0.*

Exercise 4. *Prove that the Knapsack auction allocation rule x^G induced by the greedy $(1/2)$-approximation algorithm is monotone.*

*Information about the course can be found at http://kam.mff.cuni.cz/˜ryzak/