Overview

- Standard units,
- Pointers.

Standard units

Turbo Pascal is equipped with several standard units:

- crt,
- dos,
- graph,
- printer,
- ...

Units may differ for individual compilers!

Unit crt

- Unit for working with a keyboard and a display (colors, sounds)
- Variables: `LastMode` (says what textmode was the last one used before switching graphics on),
- `TextAttr` (current attributes for displaying (text). Gets operated by `TextBackground` and `TextColor`),
- Procedure `TextBackground` sets the background color, proc. `TextColor` sets the color of foreground.
- function `keypressed` (returns `boolean` saying whether any key was pressed, `clrscr` (erases the display).

Units dos, graph a printer

- Unit dos works with files, directories, disks...
- Unit graph enables graphic mode (InitGraph, CloseGraph, GraphResult, SetColor, GetColor...).
- Unit Printer serves for printing.
- All these units consist of many functions, procedures and variables. If you want to know more, you find information in Help.

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Probably you have already seen this several times:

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program nothing;
uses crt;
...
begin
...  repeat until keypressed;
end.
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Use of unit `crt`, namely its function `keypressed`.

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- if we do not know pointers.
- Memory is linearly organized (individual addresses are indexed by natural numbers usually in hexadecimal system),
- on these addresses, data (and also code) can be stored.

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- but this functionality has to be used responsibily – we are not the only ones to use the memory
- in particular we are sharing the memory with the code we are executing.
- So one has to pay attention – using pointers incorrectly can crash your program (or even your system)!!!

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- But in practice it is not so simple!

Memory organization

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- Memory contains code, static data, buffer and a heap.
- Where does a pointer point to?
- A correct pointer should point into the heap, but an incorrectly created pointer can point anywhere!

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- Also it may happen that several pointers are pointing at the same point (pointer-aliasing).

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- Otherwise we create a memory leak!

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- `writeln(a^);` – what does this do?

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- Some languages use a garbage-collector (Java, C#), i.e., no explicit deallocation is necessary, garbage-collector takes effect at unexpected time (convenient but not as efficient as explicit deallocation).
- Pascal does not have a garbage-collector.

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- Applications: Library, phone-book,...
- Individual elements are pointing at their ancestors.
- How do we recognize the end?
- By a special constant `nil` (representing address 0).

Example

```
type ll=^packet;
     packet=record
          data:integer;
          next:ll;
     end;
var list,tmp:ll;
```

Linked list of numbers – read and write

```
begin list:=nil; tmp:=nil;
      while not EOF do
      begin new(tmp);
            readln(tmp^.data);
            tmp^.next:=list;
            list:=tmp;
      end;
      while list<>nil do
      begin writeln(list^.data);
            tmp:=list;
            list:=list^.next;
            dispose(tmp);
      end;
end.
```

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- without head/tail
- bidirectional (pointers `next` and `prev`).

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- It is possible to implement them using array,...
- but it is much better to use linked lists!

Buffer

Implementation I/III

```
type pbuf=^buf;
buf=record
     val:integer;
     next:pbuf;
end;
var head:pbuf;
procedure init;
begin head:=nil;
end;
```

Buffer

Implementation II/III

```
type pbuf=^buf;
buf=record
     val:integer;
     next:pbuf;
end;
var head:pbuf;
procedure push(what:integer);
var tmp:pbuf;
begin
     new(tmp);
     tmp^.val:=what;
     tmp^.next:=head;
     head:=tmp;
end;
```

Buffer

Implementation III

```
function pop:integer;
var tmp:pbuf;
begin
     tmp:=head;
     if head<>nil then
     begin pop:=head^.val;
           head:=tmp^.next;
           dispose(pom);
     end else
     begin writeln('Error!');
           pop:=-1;
     end;
```

Queue

Implementation

```
type=pq=^queue;
queue=record
     val:integer;
     next:pq;
end;
var head,tail:pq;
procedure init;
begin
     head:=nil;      tail:=nil; end;
```

```
procedure enqueue(what:integer);
var tmp:pq;
begin if head=nil then
     begin new(head);
          tail:=head;
          head^.next:=nil;
          head^.val:=what;
     end else
     begin new(tmp);
          tmp^.next:=nil;
          tmp^.val:=what;
          head^.next:=tmp;
          head:=tmp;
     end;
end;
```

```
function dequeue:integer;
var tmp:pq;
begin if head=nil then
      begin dequeue:=-1;
      end else
      begin if head=tail then
            begin dequeue:=tail^.val;
                  dispose(tail);
                  head:=nil; tail:=nil;
            end else
            begin dequeue:=tail^.val;
                  tmp:=tail;
                  tail:=tail^.next;
                  dispose(tmp);
            end;
      end;
end;
```

Switch two neighboring elements

Switch an element in a linked list with its neighbor

```
procedure swap(var head:ll;what:ll);
var tmp:ll;
begin tmp:=head;
      if head=what then
      begin head:=head^.next;
            tmp^.next:=head^.next;
            head^.next:=tmp;
      end else
      begin while(tmp^.next<>what) do
                  tmp:=tmp^.next;
            tmp^.next:=what^.next;
            what^.next:=tmp^.next^.next;
            tmp^.next^.next:=what;
end; end;
```

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- Good exercise: Bubblesort over linked list.
- Organizing (an ordered) linked list (functions insert, delete and member that are working with the ordered linked list).

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- Example – see webpage (or we are going to write it here).

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- Move-front rule, transposition rule:
- When accessing a member, we move it to the beginning or change with its (immediate) predecessor, respectively.
- Idea: Usually we are accessing the same element repeatedly (in a short time) but our interests are changing.

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- How does one implement this?
- Each element gets more than one sibling (left, right).

Tree representation

in Pascal

```
type tree=^vertex;
     vertex=record
          val:longint;
          left:tree;
          right:tree;
          ...
     end;
```

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- If we build it badly, it collapses into a linked-list.
- How to build a balanced binary search tree (and how to keep the tree balanced)?
- A balanced BST is a tree where for each element # elements in the left subtree (of this element) and # elements in the right subtree differ at most by 1.

Building a balanced BST

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- Build a balanced BST on smaller elements (recursively),

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- build a balanced BST on larger elements (recursively),
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BST – data structures

- The data is given in an array and we convert it to a tree (we omit the details of array handling).
- The following dynamic data structure represents the vertices of a tree:

```
type pbst:^bst;
     bst=record
          val:longint;
          left:pbst;
          right:pbst;
```

Building a balanced BST
(pseudocode)

```
function build(array):pbst;
begin
    if empty(array) then build:=nil; else begin
        med:=median(array);
        small:=smaller(med,array);
        large:=larger(med,array);
        new(root);
        root^.hod:=med;
        root^.left:=build(small);
        root^.right:=build(large);
        build:=root;
    end;
end;
```

Further operations on balanced BST

member, insert, delete

- Operation member is simple:

```
function member(what:longint,where:pbst):pbst;
begin if where=nil then member:=nil
      else  if where^.val=what then member:=where
            else  if where^.val>what then
                        member:=member(where^.left)
                  else  member:=member(where^.right);
end;
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- Beware of the algorithm's implicit assumption using trichotomy (i.e., the third branch ensures that where^.val<what)
- Function insert and delete are almost unimplementable (it would be necessary to destruct the whole tree).

Binary search tree

far from being balanced!

```
procedure insert(what,where);
begin {Marginal cases!}
      while((( what<where^.val) and
(where^.left<>nil)) or
            ((what>where^.val)and
(where^.right<>nil)))
            if(what<where^.val) then
where:=where^.left
            else  where:=where^.right;
      if(what=where^.val) then error("Already
there!");
      if(what<where^.val) then
      begin new(where^.left);
            kam:=where^.left;
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  now the erased element behaves as with out-degree 1.

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- In a short time the tree looks like a linked list.

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- Now, the deleted vertex (on the incorrect location) has an out-degree at most 1 $\Rightarrow$
- delete it (bypass).
- Instead of the left-most element in the right subtree we may use the right-most element in the left subtree (as it has the closest value to the erased element). Thus both keep the pivoting properties of the erased element.

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- after `insert` and `delete` we perform the balance-renewing operations.
- For each vertex we define a value `balance` saying `depth_right - depth_left`, permitted values are `-1`, `0` and `1`.

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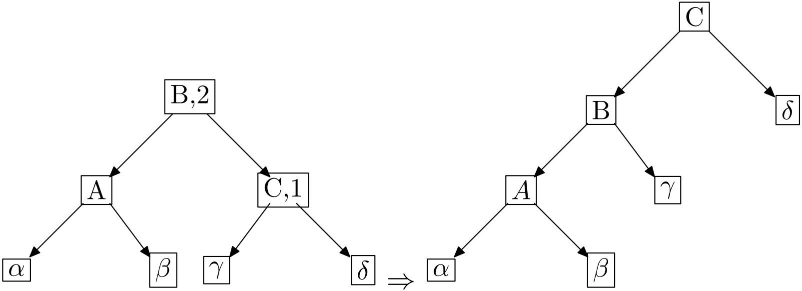
- The tree may be falling "to the side" or "to the interior".
- In the former case we use a rotation, in the latter a double-rotation.

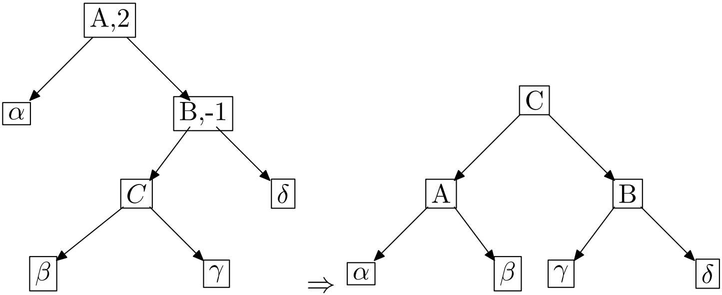
Rotation

Tree is falling "to the side".



Double-rotation

Tree is falling "to the interior".



Analysis and remarks

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FIXME!!!

binary search trees,
AVL-trees,
red-black-trees.

FIXME

Passing a function as an argument.
A queue and a buffer, graph-searching algorithms (including graph representation). Odstrasujici priklady (slidy10.tex for mathematicians).