

# Overview

- Simple sorting algorithms,
- Units,
- Pointers.

# Sorting – the motivation

- We have read the data,
- we want to process it in a monotone ordering.
- How to do that? Sort, process.
- Let us assume that the data has been read into an array.

# The problem of sorting – simple sorting algorithms

- BubbleSort,
- InsertSort,
- SelectSort,
- QuickSort.

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- Individual elements are "bubbling" in the right direction.
- We iterate this process until no swap takes place.

# Bubblesort in pseudocode

- weswapped:=true;
- while weswapped do  
begin
  - for i:=1 to length - 1 do  
begin
    - weswapped:=false;
    - if numbers[i]>numbers[i+1] then  
begin swap(numbers[i],numbers[i+1]);  
weswapped:=true;
    - end;
  - end;
- end;



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- Thus altogether  $O(n^2)$ .
- We can also implement the algorithm so that in odd iterations we bubble from left to right and in even iterations from right to left. This is called **Shakesort**. Its complexity is the same.

# Insert- and Select-sort

Selectsort:

- Repeat until the array to sort is empty:
- Find a minimum in the array to sort and add it to the sorted array.

Insertsort:

- Repeat until the array to sort is empty:
- Take the first element of the array to sort and place it onto the correct position in the target array, i.e.:  
find the position where this element should be in the target array, add it there and the rest of the target array move one position further.

Complexity-analysis: We iterate the process  $n$  times. One iteration takes at most  $cn$  steps (for some constant  $c$ ). Therefore altogether  $O(n^2)$ .

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- Employ recursion on  $B$ , employ recursion on  $C$
- Output the array  $B$ , output pivot  $p$  (as many times as it was in  $A$ ), output  $C$ .

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- To improve this algorithm we want to find a median - but we have to do it in linear time.

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- This method is specific by dividing the data in a fixed way, e.g., Quicksort.
- Technically we are designing recursive algorithms with complexity  $T(n) = \sum_{i=1}^k T(n_i)$  where  $\sum_{i=1}^k n_i = n$ .

# FIXME!!!

Here should be a quicksort implementation!

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- We may copy (click'n'paste) them into the other source files (bad idea)
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- The latter approach is referred as the **units**.

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- Source code gets spreaded into several files,
- it is not necessary to store the code more than once when we want to share it in several projects.

# Units – syntax and semantic

- Instead of with the keyword `program`, we start such files with the keyword `unit`,
- after this keyword we place the name of the unit. Please, note that the name must correspond with the filename. Also the keyword `unit` is compulsory.
- A unit consists of an `interface` (what's visible from the outside)
- and of `implementation` (internal part where the interface is implemented).

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- Interface consists of:
  - variable definitions (when the variables should be publicly visible),
  - function (and proc.) prototypes (when the function should be publicly visible),
  - prototype is the header of the function, i.e., the "first line".

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- variable definitions (for internal variables of the unit),
- definition of any stuff that should be (publicly) invisible,
- definition of internal functions (not mentioned in interface).
- We finish the unit by keyword `end`. (followed by full-stop)

# Units – example

```
unit sorting;
interface
    type po=array[0..9] of integer;
    procedure bubble(var arr:array of integer);
    procedure select(var a:po);
    procedure insert(var a:po);
    procedure quicksort(var arr:array of
integer;number:integer);
    procedure output(a:array of integer);
```

## Units – example (cont.)

```
...
implementation
  var inserted:integer;
  procedure bubble(var arr:array of integer);
    ...
    function extract_min(var a:po):integer;
    {This function will not be visible from
outside!}
    ...
    procedure select(var a:po):integer;
    ...
    ...
end.
```

# Units – how to use them

- When using a unit, we announce it with a keyword `uses` followed by the name of the unit:
- Example: `uses sorting;`

## Using the unit – example

```
program sort;
uses sorting;
var p:array [0..9] of integer;
    i:integer;
begin
    for i:=0 to 9 do
        read(p[i]);
    quicksort(p,1,10);
    output(p);
end.
```