Introduction to approximation and randomized algorithms

5th exercise session 11th December 2019

Exercise 1. Consider the graph balancing problem where we get an undirected graph G with nonnegative edge weights w_e. Our goal is to orient the edges in such a way the maximum load of a vertex (the sum of weights of edges directed to the vertex) is minimized. In other words, we get an undirected graph $G = (V,E)$ and we want to turn it into a directed graph $G' = (V,E')$ by choosing one of the two possible directions for each edge such that we minimize $\max_{v \in V} \sum_{(u,v) \in E'} w_{uv}$. Formulate this problem as an integer program, find its linear relaxation and construct a 2-approximation algorithm.

Exercise 2. Now we'll look at the Max Dicut problem. In this problem we get a directed graph $G = (V,E)$ with nonnegative weights and we want to find a set $S \subseteq V$ such that the sum of weights of edges going from S to $V \setminus S$ (only in this direction) is maximized.

a) First find a 4-approximation probabilistic algorithm.
b) Then try to find a better algorithm based on rounding of linear relaxation. Formulate this problem as a integer program with variables x_v for vertices and y_{uv} for edges. Then suppose we have an optimal solution x^*,y^* of a linear relaxation of this integer program. We'll construct the set S such that for every vertex v we put it inside with probability $\frac{1}{4} + \frac{x_v^*}{2}$. Show that this algorithm is a 2-approximation.

Exercise 3. Let us look at the Maximum Coverage problem which is a problem similar to a Set Cover problem. On input we get universe U, sets $S_1, \ldots, S_m \subseteq U$ and a number $k \in \mathbb{N}$. Our goal is to select k sets from $S_1, \ldots, S_m$ such that the number of covered elements from U is maximized. Construct and analyze an algorithm based on rounding of linear relaxation. What approximation ratio do we get?