# Syllabus

February, 2006

A $*$ denotes a theorem which was stated, but not proved.

## 1

Characters; concepts of convergence ($\sigma_n$, normed, $S_n$); Cesaro means. Locally compact, Abelian (LCA) groups; some examples: $\mathbb{Z}_2$, $\mathbb{Z}_2^n$, $\mathbb{R}$, $\mathbb{T}$.

Fejer's theorem: If $f : \mathbb{T} \to \mathbb{C}$ is Riemann integrable, and $f$ is continuous at $t$, then $\lim_n \sigma_n(f,t) = f(t)$. Further, if $f$ is continuous, then the convergence is uniform.

$*$Kolmogorov's theorem: trigonometric polynomials are dense in continuous functions with the $L_\infty$ norm.

Weierstrass's theorem: polynomials are dense in continuous functions with the $L_\infty$ norm.

## 2

Chebyshev polynomials and their $*$minimality (in $L_\infty$ norm.) The $*$Hausdorff moment problem.

$*$Carleson's theorem: if $f$ is Riemann integrable, then $S_n(f,t) \xrightarrow{n} f(t)$ a.e. $*$Kahane and Katznelson's converse: if $E$ is a measure-zero set, then there is an integrable $f$ such that $S_n(f,t) \xrightarrow{n} f(t)$ except on $E$.

$L_2$ theory; the Riemann-Lebesgue lemma; convergence theorems: if $f \in C^1$ then $S_n(f,t) \to f$ uniformly.

The Bessel inequality. Parseval's theorem: $< f, g >=< \hat{f}, \hat{g} >$.

## 3

The convolution theorem: $\widehat{f * g} = \hat{f}\hat{g}$. A sketch of Steiner's "solution" to the Greek isoperimetric problem; Hurwitz's proof. The edge-isoperimetric inequality on the discrete cube.

## 4

Linear programming; the inclusion-exclusion principle. The Linial-Nisan result on approximation: if $P(\cap_{i\in S}A_i) = P(\cap_{i\in S}B_i)$ for all $|S| \le k$, then $|P(\cup A_i) - P(\cup B_i)| \le \exp(\Theta(k/\sqrt{n}))$.

Codes: distance/radius of a codes, the MacWilliams identity: $P_C(x, y) = |C|P_{C^\perp}(y - x, y + x)$. The Hadamard code, linear codes, the orthogonal code.

## 5

Bit rate of a code; the Gilbert-Varshanov bound: $R(\delta) \ge 1 - H(\delta)$; the Elias bound: $R(\delta) \le 1 - H\left(\frac{1-\sqrt{1-2\delta}}{2}\right)$; the Delsarte linear programming problem; Krawtchuk polynomials; the sphere-packing bound: $R(\delta) \le 1 - H(\delta/2)$.

## 6

Influence of variables; the tribes function; the KKL theorem: there is a variable with influence $\mathbb{E}(f)\mathbb{E}(1 - f)\Omega(\log(n)/n)$; the Bonami-Beckner hypercontractive inequality: $|T_\epsilon(f)|_2 \le |f|_{1+\epsilon^2}$; $*$BKKKL (KKL for the solid cube); the measure $\mu_p$; sharp thresholds.

## 7

The correlation inequality ($\mu(fg) \ge \mu(f)\mu(g)$ in the monotone case); Kleitman's theorem: given a collection of size $\sum_{i=0}^{r} \binom{n}{i}$ of $n$-bit strings, there are two with Hamming distance $2r$; $*$Talagrand's theorem for monotone functions; Russo's lemma—$\mathrm{d}\mu_p(f)/\mathrm{d}p = \sum_i \mathrm{Inf}_i(f)$ (proved in exercise); $*$Erdös-Rényi result: connectivity has a sharp threshold; transitive graphs and graph properties; duals of Boolean functions; tribes and antitribes.

## 8

$*$Edge and vertex boundaries; the $*$Kruskal-Katona theorem: lexical ordering provides minimal shadow; canonical paths; first-passage percolation; sharp thresholds for graph properties.