Exercise 8

February, 2006

1. Prove directly (i.e., without using Kolmogorov's zero-one law) the result of question 4 in the previous exercise: that percolation in $\mathbb{Z}^2$ has a critical probability p_c.

2. What is the expectancy of the majority function on $2n+1$ inputs if each input has probability p of being 1?

3. (Beginning of Friedgut's proof of BKKKL) Let $f : [0,1]^n \to \{0,1\}$ be monotone. Divide the cube into k^n subcubes, such that each subcube has an edge of length $1/k$, in the obvious way. Prove that f is nonconstant on at most nk^{n-1} subcubes.

4. Let $f : \{0,1\}^n \to \{0,1\}$ be given. Define a probability space as follows: given an input vector x, we modify each bit of x with probability ϵ. We then plug in the result to f. The function $f_\epsilon(x)$ is the expectancy of the result. What is the Fourier expansion of f_ϵ?

5. Prove that $T_{\epsilon_1} T_{\epsilon_2} = T_{\epsilon_1 \epsilon_2}$.