Combinatorics

Exercise 9 – Projective planes and BIBD's

Homework

Deadline: **21. 12. 2022**
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1. A *finite affine plane* is a set system $\mathcal{A} = (X, \mathcal{L})$ satisfying the following axioms:

   A1 For every $L \in \mathcal{L}$ and every $x \in X \setminus L$ there is unique $L' \in \mathcal{L}$ such that $x \in L'$ and $L' \cap L = \emptyset$,

   A2 for every two distinct $x, y \in X$, there is unique $L \in \mathcal{L}$ such that $x, y \in L$,

   A3 for every $L \in \mathcal{L}$ it holds that $|L| \geq 2$, and

   A4 there exists distinct $x, y, z \in X$ such that $\{x, y, z\} \not\subseteq L$ for every $L \in \mathcal{L}$.

   (a) Prove that if one removes one line and all of its points from a projective plane, one gets an affine plane. **[1 point]**

   (b) Prove that if $\mathcal{A}$ is an affine plane, then one can add $n + 1$ points and 1 line (and extend the existing lines by some of the new points) to get a projective plane. **[1 point]**

Problems

1. Recall the axioms of projective planes from the lecture and prove that if (A1) and (A2) hold, then (A3) is equivalent to (A3').
2. Draw the projective plane of order 3. (The aim of this is that you get hands-on experience with the general construction of projecctive planes, it might be useful e.g. at the exam.)
3. Let q be a prime and n, d positive integers. Put $V = GF(q)^n$ (as a vector space) and let $\mathcal{B}$ consist of all d-dimensional subspaces of V. Find v, k, λ such that $(V, \mathcal{B})$ is a (v, k, λ)-BIBD.