Combinatorics

Exercise 3 – Tutte theorem

Problems

1. Show that Tutte theorem implies Hall's theorem (the difficult implication).
2. Prove that every 3-regular bridgeless graph has a perfect matching.
3. For every $k \geq 1$, build a graph with minimum degree at least k and exactly one perfect matching.
4. Prove that if G has no perfect matching, then G has a vertex v such that all edges incident with v belong to some maximum matching (each to a different one of course).
5. Construct a 3-regular graph with no perfect matching.