Kombinatorika a grafy 1

Cvičení #12 – Q & (snad) A

Příklady

Příklady jsou pro zabavení těch, kteří přišli, přestože žádné otázky ani odpovědi nemají.

1. Dokažte, že Hadamardovy kódy vzniklé Sylvestrovou konstrukcí jsou lineární nad $\mathbb{F}_2$ (po bijekci $1 \mapsto 0$, $-1 \mapsto 1$).
2. Křemílek si do sešitu napsal čísla $1, \ldots, 500$ v nějakém pořadí. Vochomůrka se může Křemílka zeptat na libovolných 250 z nich a Křemílek mu řekne, v jakém pořadí se v sešitě nacházejí. Na kolik nejméně otázek dokáže Vochomůrka vždy určit, co přesně si Křemílek napsal do sešitu?
3. Let σ be a permutation of $\{1, \ldots, 2n\}$. We say that σ is *splendid* if there is i such that $|\sigma(i) - \sigma(i+1)| = n$, otherwise σ is *plain*. Prove that there are more splendid permutations than plain ones (without explicitly counting them if possible).
4. There are n points labelled $1, 2, \ldots, n$. Between every pair of them there is an arrow going from the smaller to the larger one. A red-blue colouring of the arrows is *delicious* if there is no pair of points A, B with both blue and red oriented paths from A to B. Prove that there are $n!$ delicious colourings (without using paper and pen if possible).
5. At the Rector's Sports Day there is a prestigious football tennis match between the students and the professors. The rules are as follows: Each time, one of the teams serves the ball and then one of the teams wins a point. The first team to reach n points wins. The professors are the first to serve and then there are two service schemes: Either the teams alternate, or the team to win the last point serves the next ball. Assuming that the probability of winning a point only depends on which team is serving, prove that the probability of winning the match is independent from the choice of the service scheme.