

Combinatorics

Exercise 8 – Kuratowski theorem

Problems

1. Let G be a vertex-3-connected planar graph with a convex embedding on the plane such that the edges of the outer face form a convex polygon. Prove that all edges of this embedding are in fact straight-line segments. Use this to show that the Convex faces theorem implies that every vertex-3-connected planar graph has a plane drawing where all edges are straight-line segments.
2. Find a planar graph with a planar embedding such that all edges are straight-line segments, but there is an inner face which is not convex.
3. Find a vertex-2-connected planar graph G , an integer k and two planar embeddings T, T' of G such that T contains a face whose boundary contains k vertices while T' does not. Conclude that the Uniqueness theorem cannot be extended to vertex-2-connected graphs.