Combinatorics
Exercise 3 – Tutte theorem

Problems

1. How many minimum vertex covers and how many minimal vertex covers does the star on n vertices have?[1]

2. Let G be a graph and let $\mu(G)$ be the size of its maximum matching. Prove that every maximal matching in a graph has at least $\frac{\mu(G)}{2}$ edges.

3. Show that Tutte theorem implies Hall's theorem (the difficult implication).

4. Prove that every 3-regular bridgeless graph has a perfect matching.

5.

   (a) How many perfect matchings does K_n have? How many of them contain a given fixed edge e?

   (b) Let n be even. Prove that every graph on n vertices with more than $\binom{n-1}{2}$ edges has a perfect matching.

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[1]Just understand and remember the difference between minimum (nejmenší) and (inclusion) minimal (minimální vzhledem k inkluzi).