

Combinatorics

Exercise 10 – Symmetric BIBD's

Problems

1. Prove that there is a unique symmetric $(7, 3, 1)$ -BIBD (up to renaming the elements), the Fano plane.
2. Prove that the complement of the Fano plane is a symmetric $(7, 4, 2)$ -BIBD.
3. Prove that all derived designs from the complement of the Fano plane are isomorphic.
4. Construct a $(3, 2, 2)$ -BIBD as a residual design from the complement of the Fano plane. (I know, very interesting problem indeed.)
5. Find a symmetric BIBD such that not all its derived designs are isomorphic. (The only way I can solve this is by googling it and finding some random-looking $0, 1$ -matrices, can you do better?)