Combinatorics

Exercise 5 – Heawood & Vizing

Problems

1. Prove that all bipartite graphs are Vizing class 1 (i.e. $\chi'(G) = \Delta(G)$).
2. Find all (finitely many) graphs G for which $\omega(L(G)) \neq \Delta(G)$.
3. Construct a sequence of triangle-free graphs whose chromatic number goes to infinity.
4. Let G be a graph and let $d\colon V(G) \to \mathbb{N}$ be a function (recall that $0 \in \mathbb{N}$). Use Edmonds' blossom algorithm to decide in polynomial time whether G contains a (non-incuded) subgraph H such that $V(G) = V(H)$ and for every $v \in V(H)$ it holds that $\deg_H(v) = d(v)$.
5. Show that for every (orientable) surface there are only finitely many mutually non-isomorphic 7-regular graphs with a drawing on that surface.
6. Use the four color theorem to show that for every planar vertex-2-connected 3-regular graph G it holds that $\chi'(G) = 3$.