# Combinatorics <br> Exercise 4 - Kuratowski theorem 

## Homework

Deadline: 2. 11. 2020 14:00
Send to: matej@kam.mff.cuni.cz (in PDF, preferably $\mathrm{EAT}_{\mathrm{E}} \mathrm{X}$, of course)

1. Let $G$ be a vertex-3-connected planar graph with a convex embedding on the plane such that the edges of the outer face form a convex polygon. Prove that all edges of this embedding are in fact straight-line segments. Use this to show that the Convex faces theorem implies that every vertex-3-connected planar graph has a plane drawing where all edges are straight-line segments.
2. Find a planar graph with a planar embedding such that all edges are straight-line segments, but there is an inner face which is not convex.
3. Find a vertex-2-connected planar graph $G$, an integer $k$ and two planar embeddings $T, T^{\prime}$ of $G$ such that $T$ contains a face whose boundary contains $k$ vertices while $T^{\prime}$ does not. Conclude that the Uniqueness theorem cannot be extended to vertex-2-connected graphs.
