

# Combinatorics

## Exercise 2 – Tutte theorem

### Problems

1. How many minimum vertex covers and how many minimal vertex covers does the star on  $n$  vertices have?<sup>1</sup>
2. Let  $G$  be a graph and let  $\mu(G)$  be the size of its maximum matching. Prove that every maximal matching in a graph has at least  $\frac{\mu(G)}{2}$  edges.
3. Show that Tutte theorem implies Hall's theorem (the difficult implication).
4. Prove that every 3-regular bridgeless graph has a perfect matching.
5.
  - (a) How many perfect matchings does  $K_n$  have? How many of them contain a given fixed edge  $e$ ?
  - (b) Let  $n$  be even. Prove that every graph on  $n$  vertices with more than  $\binom{n-1}{2}$  edges has a perfect matching.

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<sup>1</sup>Just understand and remember the difference between minimum (nejmenší) and (inclusion) minimal (minimální vzhledem k inkluzi).