Probabilistic Techniques

Exercise #2 — Markov's and Chebyshev's inequalities

November 6, 2019

1. Let $x, y \in [0,1]$ be chosen uniformly independently. Determine $\mathbb{E}[|x-y|]$.
2. Let X be a Bernoulli random variable. Compute $\mathbb{E}X$ and $\mathrm{Var}X$. What if X was a sum of n independent identically distributed Bernoulli random variables?
3. Let n be a positive integer and $x_1, \ldots, x_n$ be independent uniformly random integers from $[-L, L]$. Prove that if $L = \frac{2^n}{400\sqrt{n}}$, there exist two subsets $I \neq J \subseteq [n]$ such that
$$\sum_{i \in I} x_i = \sum_{j \in J} x_j.$$
4. Let P be a monotone graph property, that is, a property such that if G has P then every graph H with $V(H) = V(G)$ and $E(H) \supseteq E(G)$ also has P. Prove that if $p_1 < p_2$ then
$$\Pr[G(n,p_1) \text{ has } P] \leq \Pr[G(n,p_2) \text{ has } P].$$