Friday, March 13, 2020 11:53 AM

číselné souvislé / řady

definice souřet řady

$\sum q^n$; $\sum 1/n^s$, $s>1$; $\sum 1/n = \infty$

$\left(\sum 1/n^2 = \frac{\pi^2}{6} < \infty\right)$

4a)

$\sum_{n=0}^{\infty} (-1)^n$. $(s_n) = (1, 0, 1, 1-1+1-1=0, \dots)$

$1, 0, 1, 0, 1, 0, \dots \to$ nemá limitu

$\Rightarrow \sum (-1)^n$ nekonverguje

4b)

$$\sum_{n=0}^{\infty} \frac{3^n - 2^{n+1}}{6^n} = \sum_{n=0}^{\infty} \left[\frac{3^n}{6^n} - 2 \cdot \frac{2^n}{6^n} \right] \overset{\text{fakt}}{=} \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n - 2 \cdot \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n$$

$$= \frac{1}{1-1/2} - 2 \cdot \frac{1}{1-1/3} = 2 - 2\,\frac{3}{2}$$

$= -1$

4c)

$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$

$$\frac{3+0\cdot n}{n(n+3)} = \frac{A}{n} + \frac{B}{n+3} \qquad A, B \in \mathbb{R}$$

$$= \frac{A(n+3) + Bn}{n(n+3)} = \frac{(A+B)n + 3A}{n(n+3)}$$

$\Rightarrow A+B = 0$, $3A = 3 \Rightarrow A = 1$, $B = -1$

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)} = \sum \left[\frac{1}{n} - \frac{1}{n+3}\right] = \sum_{n=1}^{\infty} \frac{1}{n} - \sum_{n=1}^{\infty} \frac{1}{n+3}$$

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \sum_{n=4}^{\infty} \frac{1}{n} \qquad \sum_{n=4}^{\infty} \frac{1}{n} = \sum_{n=1}^{\infty} \frac{1}{n+3}$$

$$= \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \cancel{\sum_{n=1}^{\infty} \frac{1}{n+3}} - \cancel{\sum_{n=1}^{\infty} \frac{1}{n+3}}$$

$$= 1 + 5/6$$