2 limsup liminf

b) $n^{\sin\left(\frac{\pi n}{2}\right)}$

$\left(\sin\left(\frac{\pi n}{2}\right)\right)_n = (0, 1, 0, -1, 0, 1, 0, -1, \dots)$

$\left(n^{\sin\left(\frac{\pi n}{2}\right)}\right)_n = (\underset{=1}{0^0}, \underset{=1}{1^1}, \underset{=1}{2^0}, \underset{=1/3}{3^{-1}}, \underset{=1}{4^0}, \underset{=5}{5^1}, 1, 1/7, 1, 9, \dots)$

$\Rightarrow$ pro n sudé je $n^{\sin\left(\frac{\pi n}{2}\right)} = 1$

pro $n \equiv 1 \mod 4$ –||– $= n$

–||– $\equiv 3 \mod 4$ –||– $= 1/n$

$\Rightarrow$ hrom. body $= \{1, +\infty, 0\}$

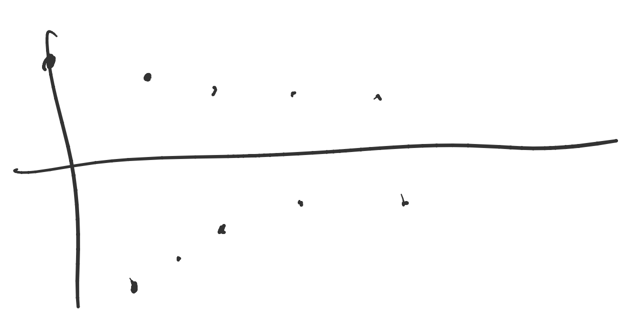
$\Rightarrow$ lim inf $a_n = 0$, limsup $a_n = +\infty$

c) $a_0 = 1000$, $a_{n+1} = \frac{3}{a_n}$

$1000, \; 3/1000, \; \frac{3}{3/1000} = 1000, \; 3/1000, \; 1000, \dots$

$$a_n = \begin{cases} 1000 & n \text{ sudé} & \in \text{hrom. body} \\ 3/1000 & n \text{ liché} & \in \text{hrom. body} \end{cases}$$

d) $a_n = (-1)^n \frac{2n+3}{n+1}$

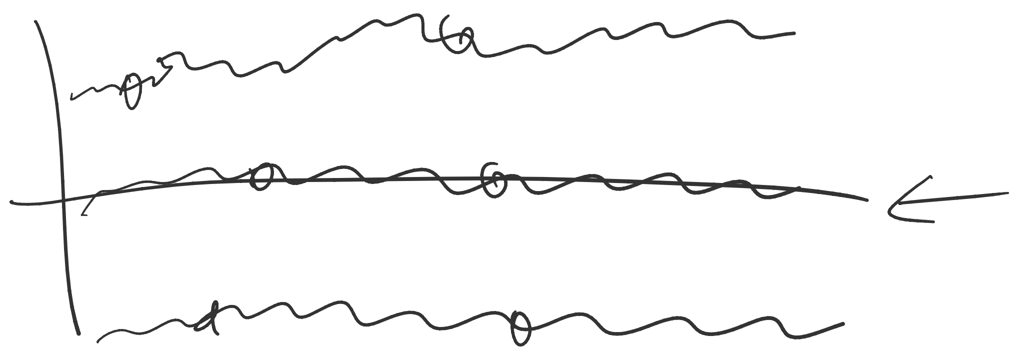


$$b_n = a_{2n} = \overbrace{(-1)^{2n}}^{1} \frac{4n+3}{2n+1} = \frac{4n+1}{2n+1} \cdot \frac{1/n}{1/n} = \underbrace{\frac{4+1/n}{2+1/n}} \to 2.$$

$$c_n = a_{2n+1} = \underbrace{(-1)^{2n+1}}_{=-1} \frac{4n+2+3}{2n+1+1} = -\frac{4n+5}{2n+2} \cdot \frac{1/n}{1/n} = -\frac{4+5/n}{2+2/n} \to -2.$$

$\Rightarrow$ hrom. body $a_n = \{-2, 2\}$

Pokud a_n má hrom. bod H
A rozdělíme "$a_n = b_n \cup c_n$"
$\Rightarrow$ b_n nebo c_n mají hrom. bod H



Def.: a_n má hrom. bod H, pokud

$\forall \varepsilon > 0 \, \forall n_0 \, \exists n \geq n_0 : |a_n - H| < \varepsilon$ $\Leftarrow$ $\forall a_n : a_n \in (b_n)$ nebo $a_n \in (c_n)$

Kdyby b_n ani c_n neměly hrom. bod H

$\Rightarrow$ $\underbrace{\exists \varepsilon > 0 \, \exists n_0 \, \forall n \geq n_0 : |b_n - H| \geq \varepsilon, \; |c_n - H| \geq \varepsilon}$

$\neg(b_n$ má hrom. bod $H) \equiv$

$\equiv \exists \varepsilon^b > 0 \, \exists n_0^b \, \forall n \geq n_0^b : |b_n - H| \geq \varepsilon^b$ $\Leftarrow$ neg. def.

$\neg(c_n$ má hr. bod $H) \equiv$

$\exists \varepsilon^c > 0 \, \exists n_0^c \, \forall n \geq n_0^c : |c_n - H| \geq \varepsilon^c$

nejsou exponenty
jsou to horní indexy.

$\varepsilon = \min(\varepsilon^b, \varepsilon^c)$ $\quad n_0 = \max(n_0^b, n_0^c)$