

Probabilistic Techniques

Exercise #1 – Expectation and the method of alteration

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Method of alteration

1. Let G be a graph on n vertices with average degree d . Then $\alpha(G) \geq \frac{n}{2d}$.
2. Let G be a graph on n vertices with minimum degree δ . Prove that G has a dominating set D with

$$|D| \leq n \frac{1 + \ln(\delta + 1)}{\delta + 1}.$$

Exercise

1. Let $A, B \in \binom{[n]}{k}$ be chosen uniformly independently randomly. Determine $\mathbb{E}[|A \cap B|]$
2. Let H be a k -uniform hypergraph with at most 2^{k-1} edges. Prove that H is 2-colourable (that is, has a colouring of vertices where no edge is monochromatic).
3. Let n be *large enough* and A be a set of n residues mod n^2 . Show that there is a set B of n residues mod n^2 such that at least half of all the residues mod n^2 can be written as $a + b$ with $a \in A$ and $b \in B$.

Hard. Let G be a graph with $m \geq 4n$. Prove that

$$cr(G) \geq \frac{1}{64} m^3 n^2.$$