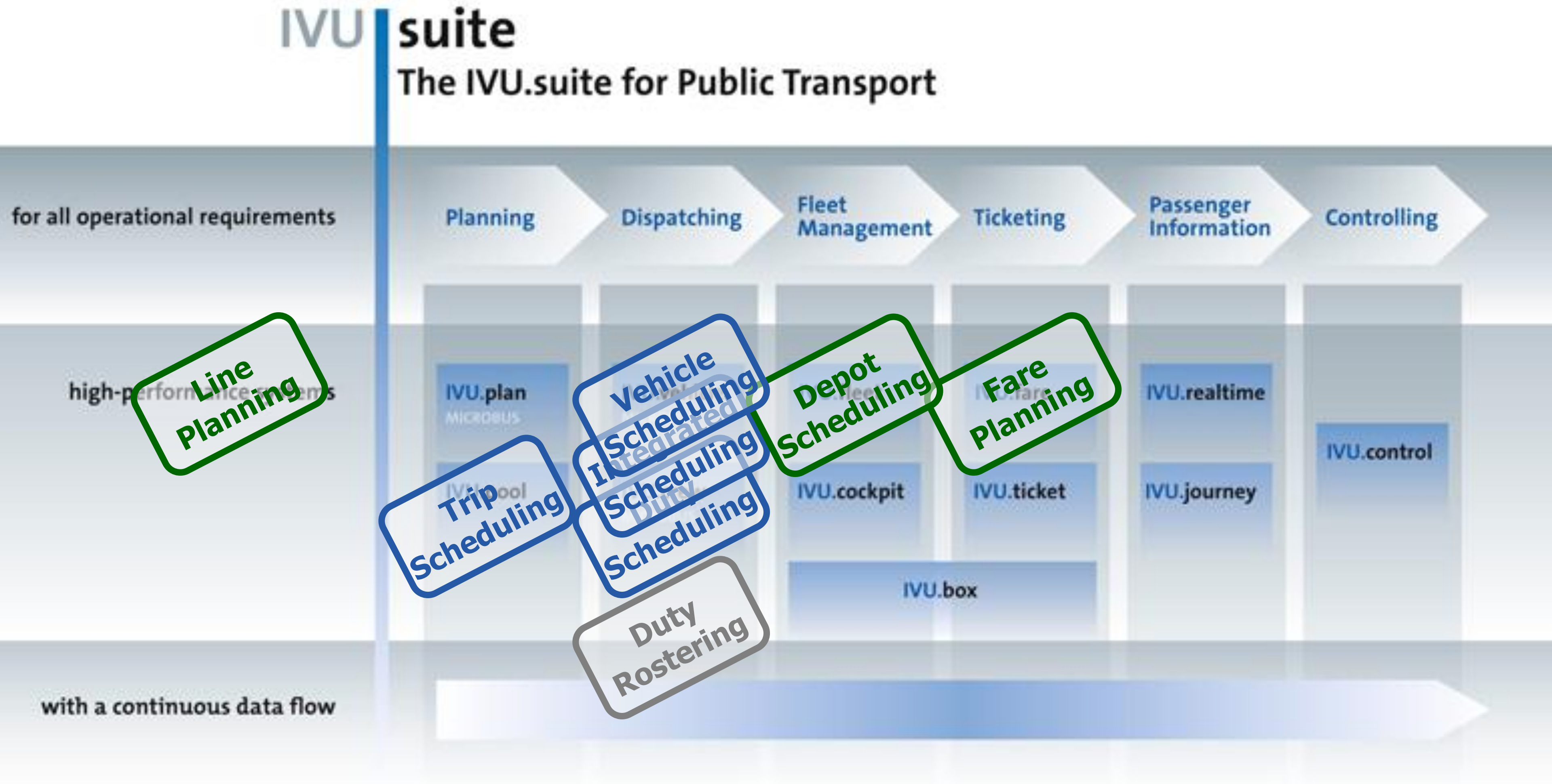
Scheduling Problems and Algorithms in Traffic and Transport

MAPSP 2011
Nymburk, 24.06.11

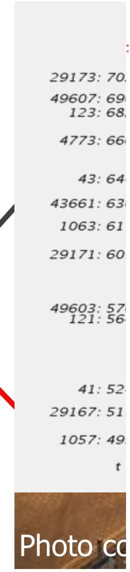
Ralf Borndörfer
Zuse-Institute Berlin

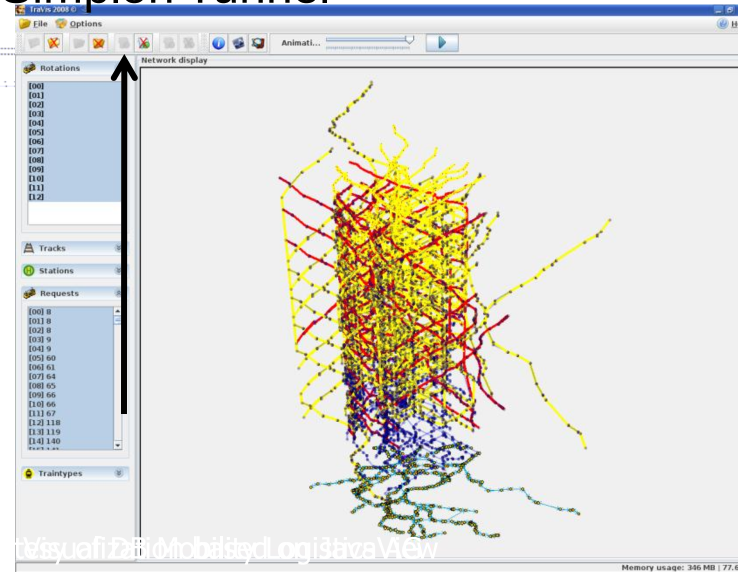
Joint work with Ivan Dovica, Martin Grötschel, Olga Heismann, Andreas Löbel, Markus Reuther, Elmar Swarat, Thomas Schlechte, Steffen Weider


IVU suite
The IVU.suite for Public Transport
for all operational requirements
Planning
Dispatching
Fleet Management
Ticketing
Passenger Information
Controlling
high-performance systems
IVU.plan
IVU.realtime
IVU.control
IVU.cockpit
IVU.ticket
IVU.journey
IVU.box
with a continuous data flow
Line Planning
Vehicle Scheduling
Integrated Scheduling
Duty Scheduling
Trip Scheduling
Depot Scheduling
Fare Planning
Duty Rostering

We want to avoid this!

Simplon Tunnel





Basic Rolling Stock Rostering Problem = Multicommodity Flow Problem

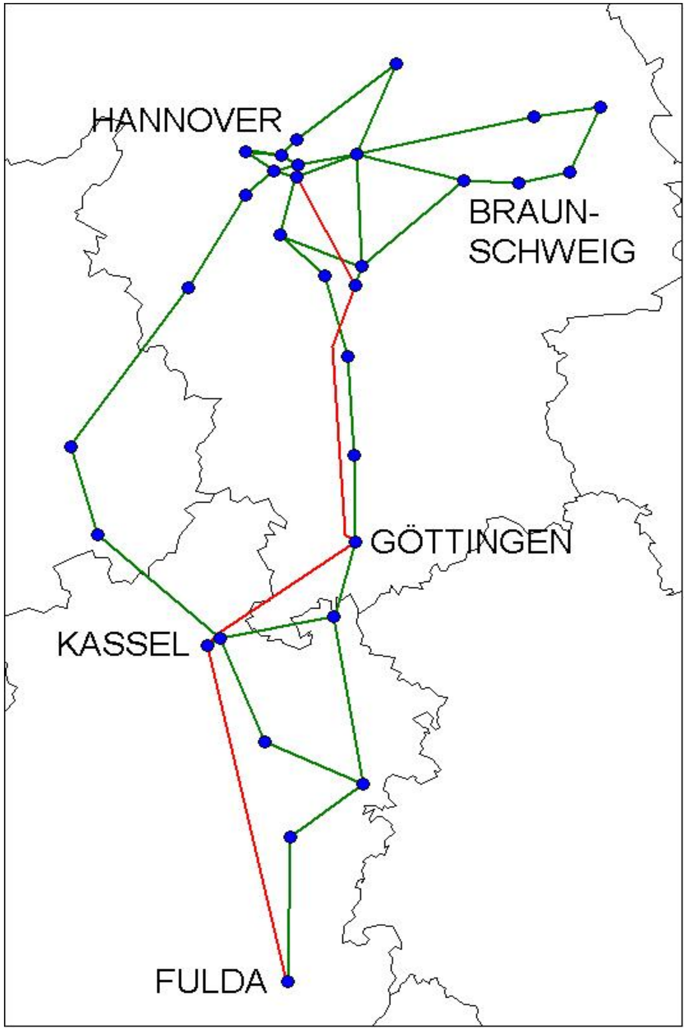
- ▷ Can be solved efficiently for networks with 10^9 arcs

Constraints complicating rolling stock rostering

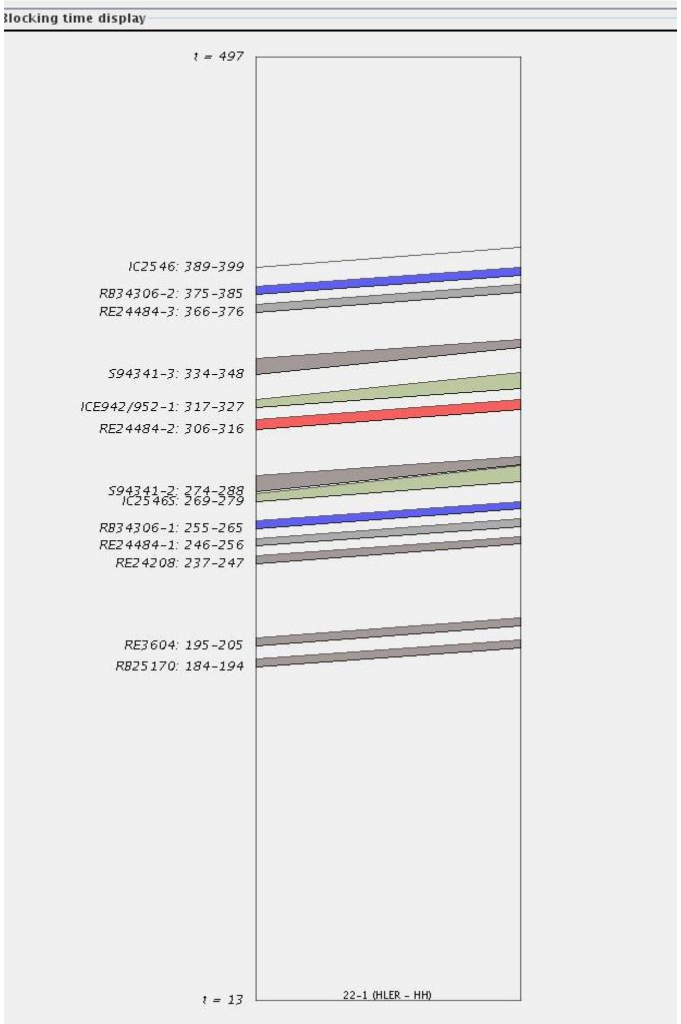
- ▷ Discretization: Space/Time ("Multiscale Problems")
- ▷ Robustness: Delay Propagation
- ▷ Path Constraints: Maintenance, Parking
- ▷ Configuration Constraints: Track Usage, Train Composition, Uniformity

Integrated Routing and Scheduling

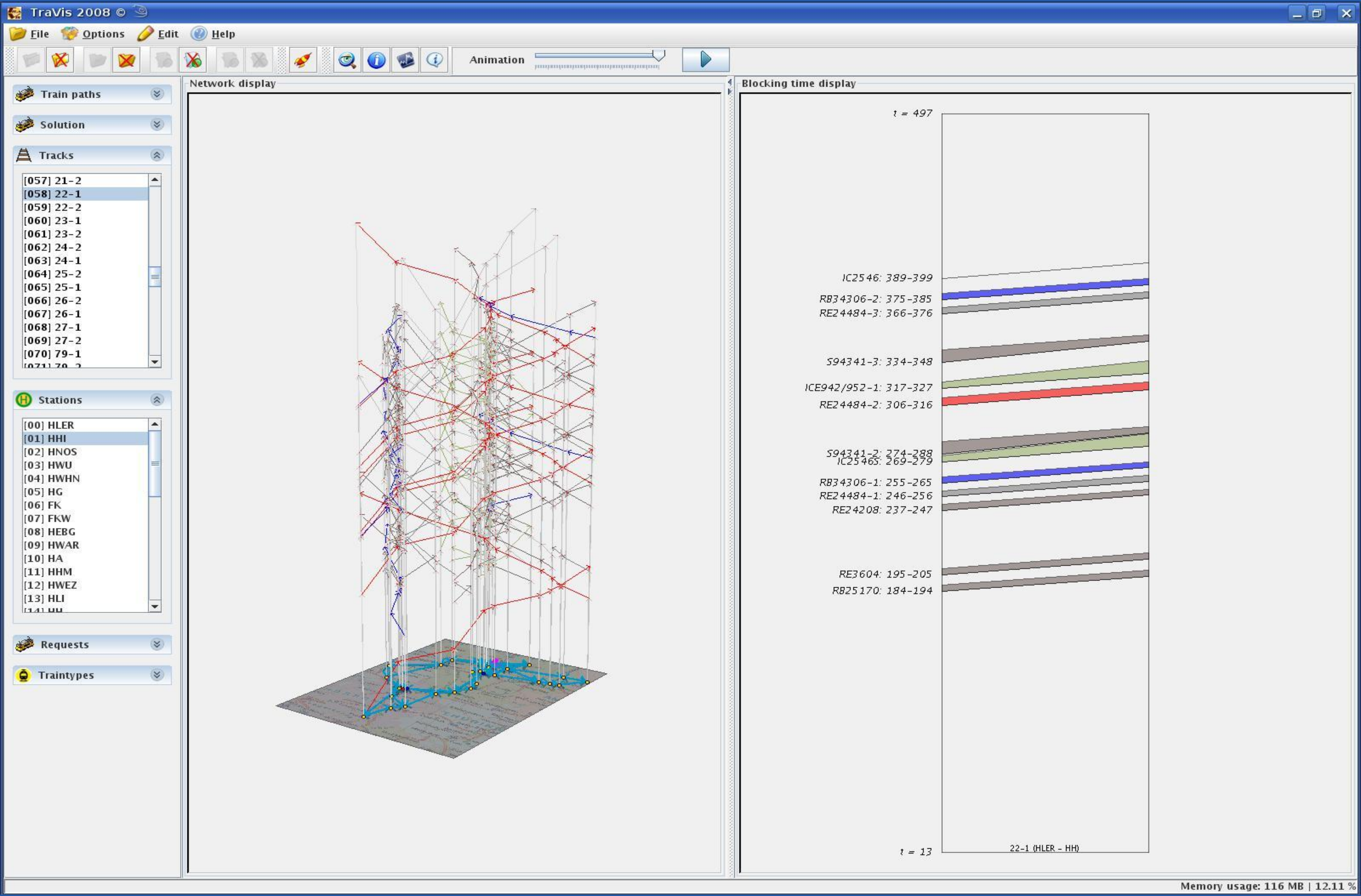
Routing



Scheduling

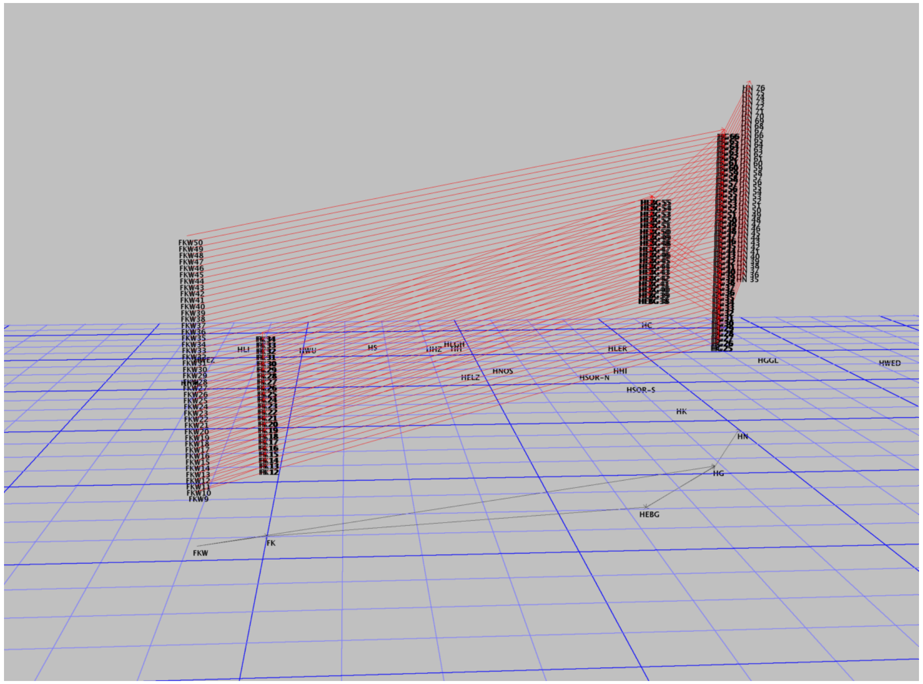


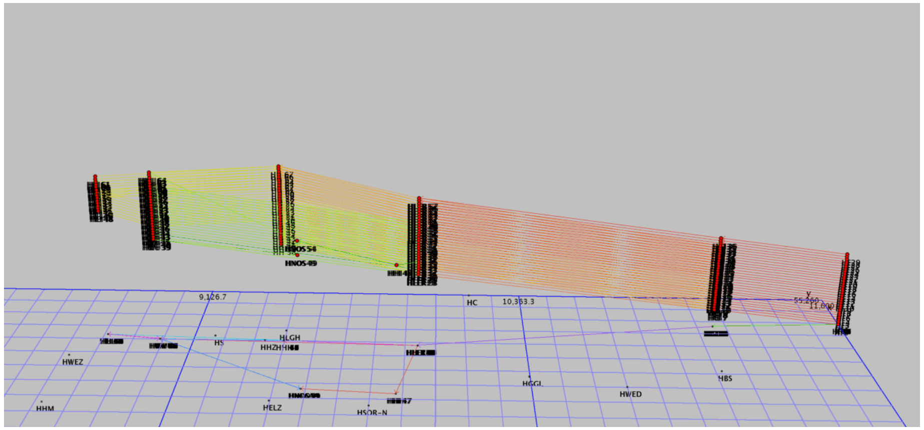
Timetable

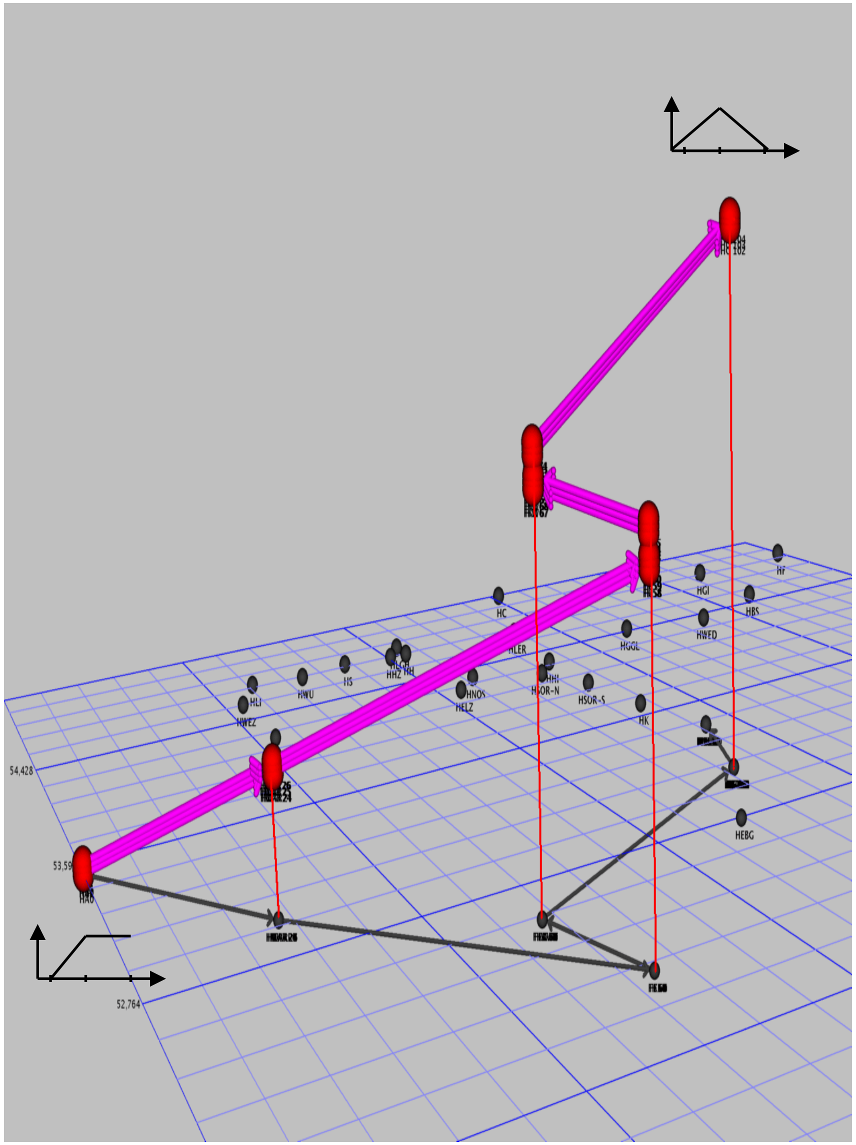


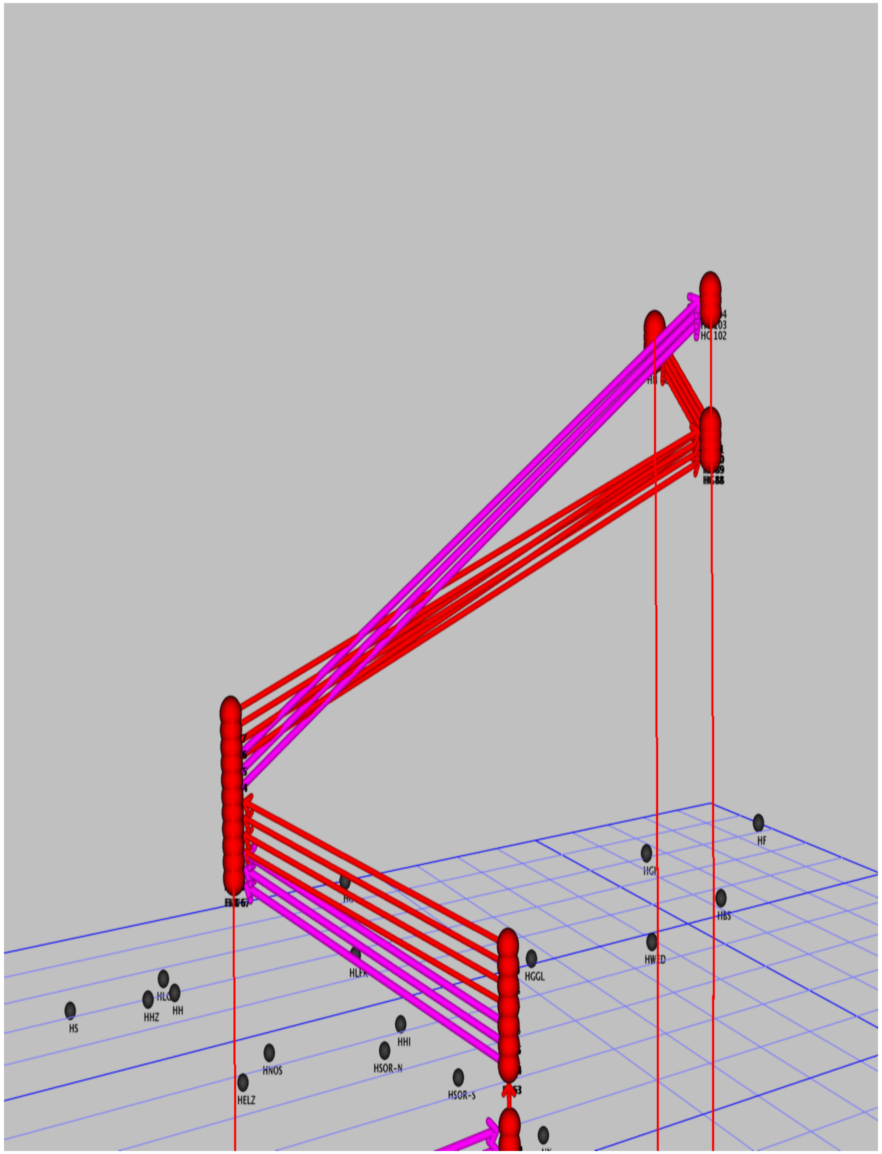
TraVis 2008 ©
File Options Edit Help
Animation
Train paths
Solution
Tracks
[057] 21-2
[058] 22-1
[059] 22-2
[060] 23-1
[061] 23-2
[062] 24-2
[063] 24-1
[064] 25-2
[065] 25-1
[066] 26-2
[067] 26-1
[068] 27-1
[069] 27-2
[070] 79-1
Stations
[00] HLER
[01] HHI
[02] HNOS
[03] HWU
[04] HWHN
[05] HG
[06] FK
[07] FKW
[08] HEBG
[09] HWAR
[10] HA
[11] HHM
[12] HWEZ
[13] HLI
Requests
Traintypes
Network display
Blocking time display
t = 497
IC2546: 389-399
RB34306-2: 375-385
RE24484-3: 366-376
S94341-3: 334-348
ICE942/952-1: 317-327
RE24484-2: 306-316
S94341-2: 274-288
IC25465: 269-279
RB34306-1: 255-265
RE24484-1: 246-256
RE24208: 237-247
RE3604: 195-205
RB25170: 184-194
t = 13
22-1 (HLER - HH)
Memory usage: 116 MB | 12.11 %

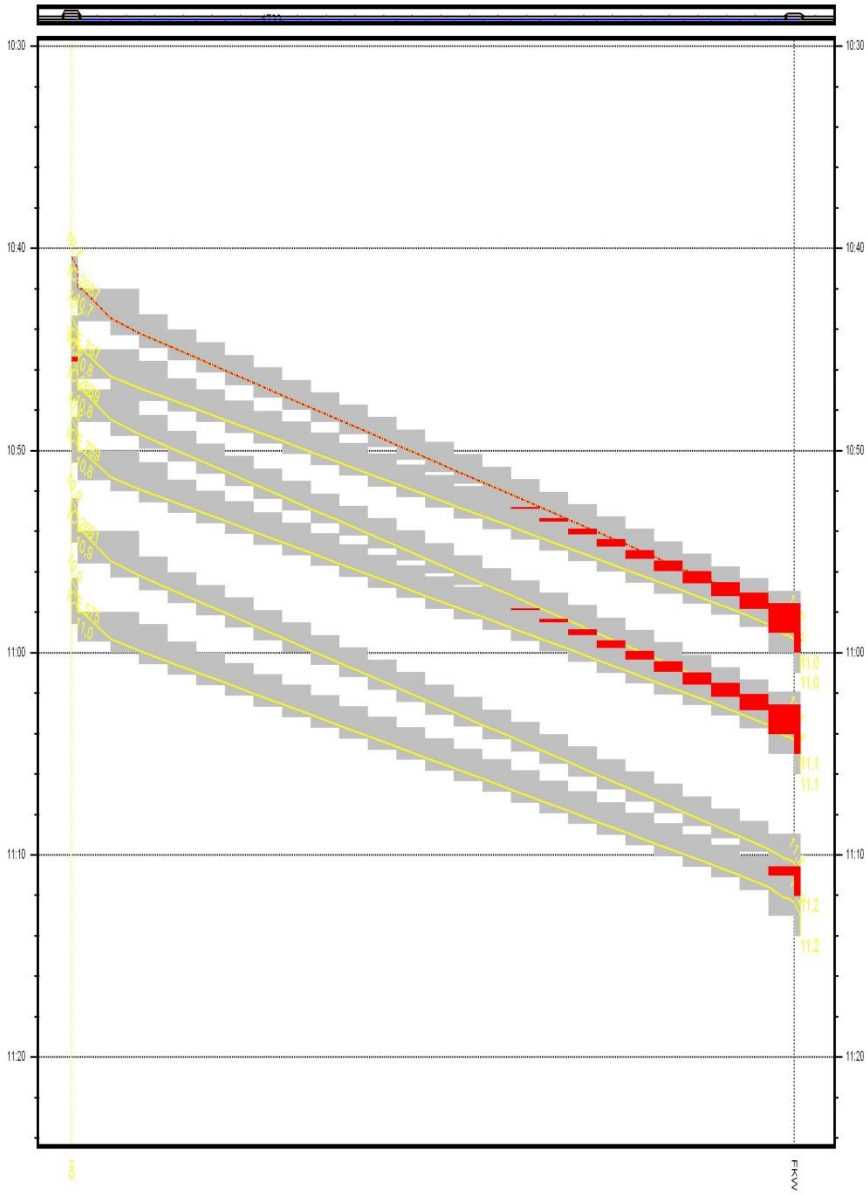
Train Routes are Flexible in Space and Time



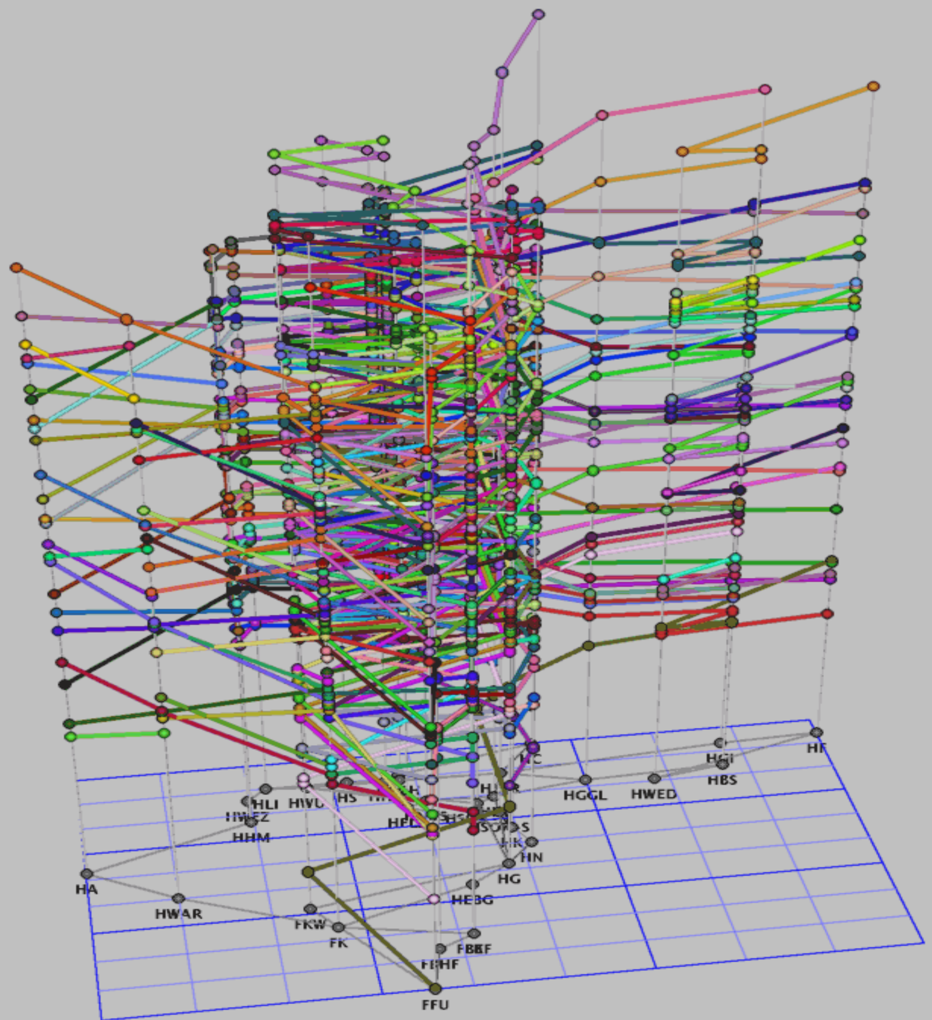




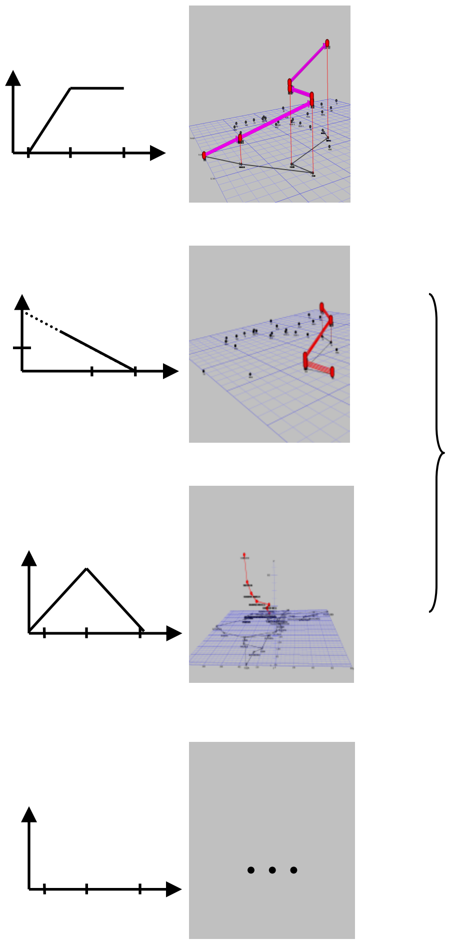

HS
HHZ
HH
HNOS
HELZ
HHI
HSOR-N
HSOR-S
HGGL
HWED
HBS
HF

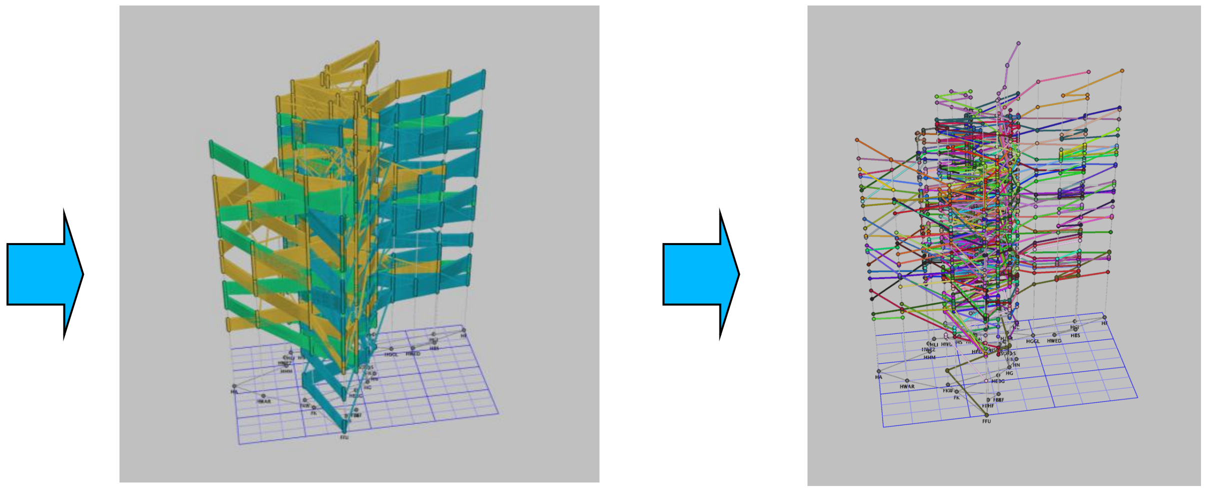

10:30
10:40
10:50
11:00
11:10
11:20
FKW

Track Allocation Graph



Track Allocation/Train Timetabling Problem

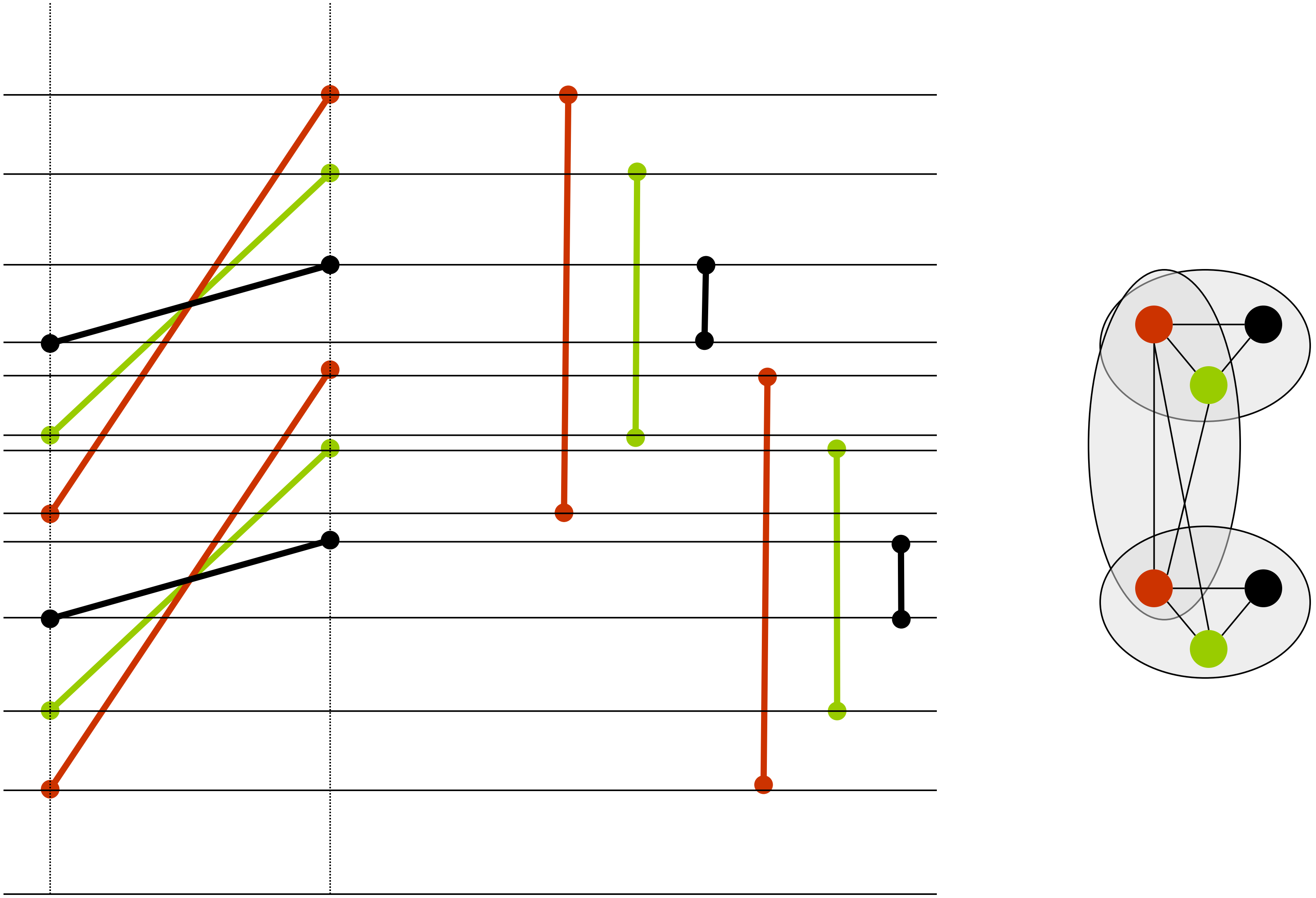




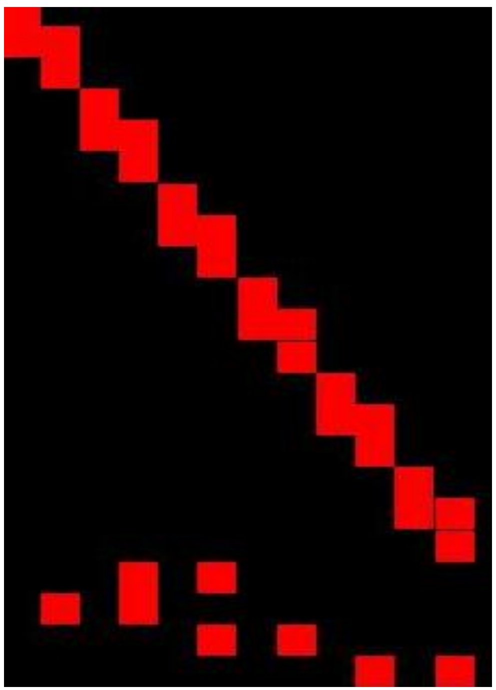
Combinatorial Optimization Problem

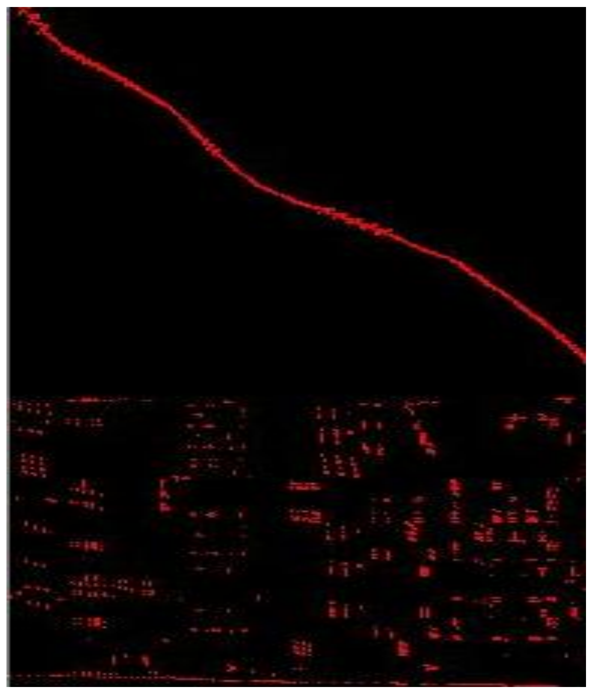
▷ Path Packing Problem

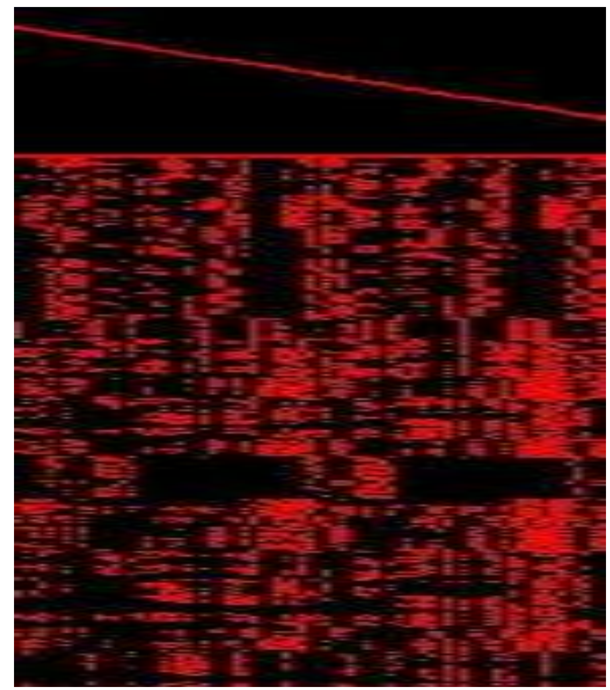
- Charnes and Miller (1956), Szpigel (1973), Jovanovic and Harker (1991),
- Cai and Goh (1994), Schrijver and Steenbeck (1994), Carey and Lockwood (1995)
- Nachtigall and Voget (1996), Odijk (1996) Higgings, Kozan and Ferreira (1997)
- **Brannlund, Lindberg, Nou, Nilsson (1998)**, Lindner (2000), Oliveira and Smith (2000)
- **Caprara, Fischetti and Toth (2002)**, Peeters (2003)
- Kroon and Peeters (2003), Mistry and Kwan (2004)
- Barber, Salido, Ingolotti, Abril, Lova, Tormas (2004)
- Semet and Schoenauer (2005),
- **Caprara, Monaci, Toth and Guida (2005)**
- Kroon, Dekker and Vromans (2005),
- Vansteenwegen and Van Oudheusden (2006), Liebchen (2006)
- **Cacchiani, Caprara, T. (2006), Cachhiani (2007)**
- Caprara, Kroon, Monaci, Peeters, Toth (2006)
- **Borndoerfer, Schlechte (2005, 2007),** Caimi G., Fuchsberger M., Laumanns M., Schüpbach K. (2007)
- **Fischer, Helmberg, Janßen, Krostitz (2008)**
- Lusby, Larsen, Ehrgott, Ryan (2009)
- Caimi (2009), Klabes (2010)
- ...

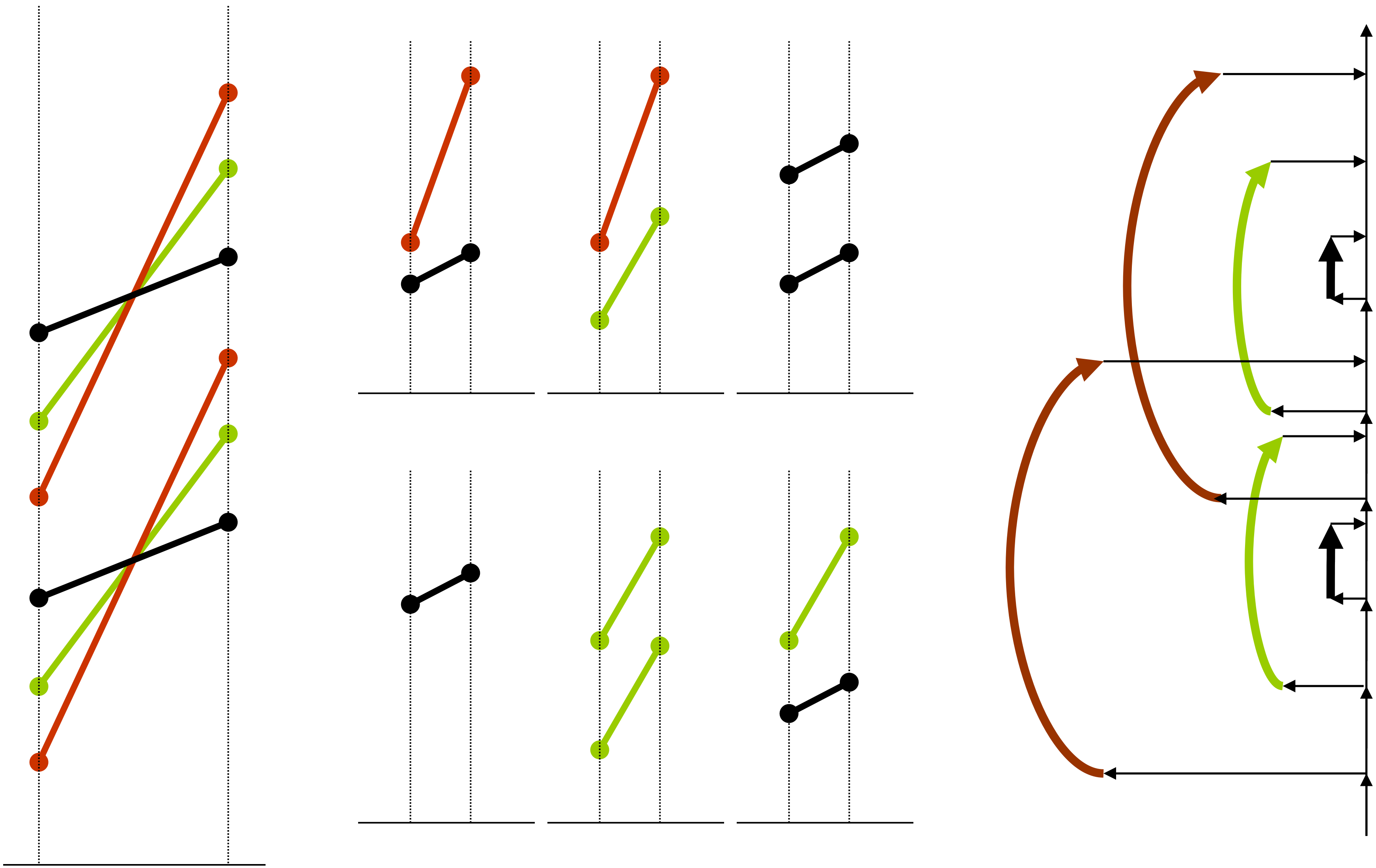


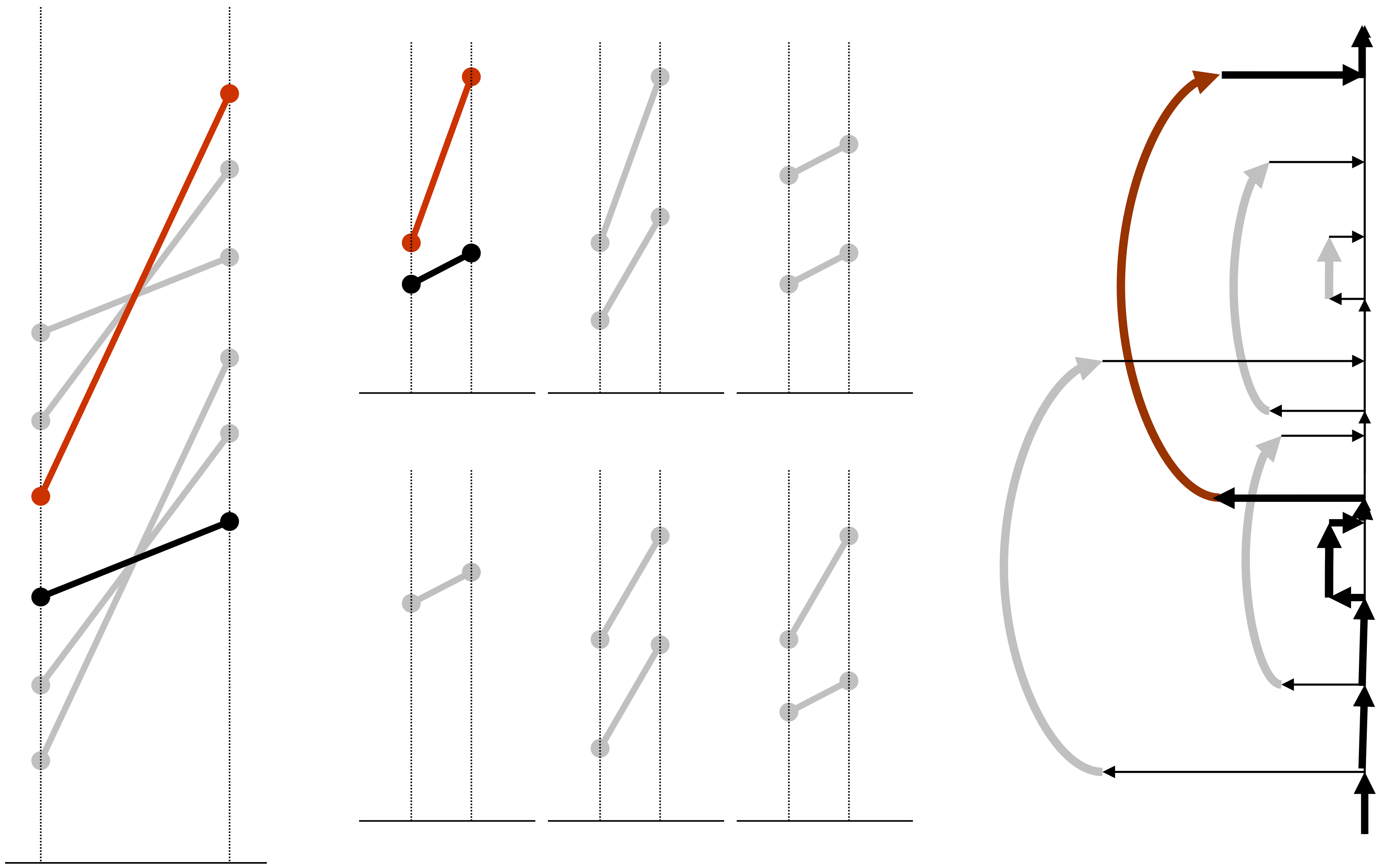
$$
\begin{array}{llllll}
\text{(APP)} & \max \sum_{i\in I}\sum_{a\in A} c_a^i x_a^i & & & & \\
\text{(i)} & \sum_{a\in\delta_i^+(v)} x_a^i - \sum_{a\in\delta_i^-(v)} x_a^i & = & \beta_i(v) & \forall v\in V, i\in I & \text{Flow} \\
\text{(ii)} & \sum_{(a,i)\in k} x_a^i & \leq & 1 & \forall k\in K & \text{Conflicts} \\
\text{(iii)} & x_a^i & \in & \{0,1\} & \forall a\in A, i\in I & \text{Integ.}
\end{array}
$$











$$
\begin{array}{llcll}
\text{(APP)} & \max \sum_{i \in I} \sum_{a \in A} c_a^i x_a^i & & & \\
\text{(i)} & \sum_{a \in \delta_i^+(v)} x_a^i - \sum_{a \in \delta_i^-(v)} x_a^i = \beta_i(v) & & \forall v \in V, i \in I & \text{Flow} \\
\text{(ii)} & \sum_{(a,i) \in k} x_a^i & \leq 1 & \forall k \in K & \text{Conflicts} \\
\text{(iii)} & x_a^i & \in \{0,1\} & \forall a \in A, i \in I & \text{Integ.}
\end{array}
$$

$$
\begin{array}{llcll}
\text{(PCP)} & \max \sum_{i \in I} \sum_{p \in P_i} \sum_{a \in p} c_a^i x_p & & & \\
\text{(i)} & \sum_{p \in P_i} x_p & \leq 1 & \forall i \in I & \text{Trains} \\
\text{(ii)} & \sum_{q \in Q_j} y_q & \leq 1 & \forall j \in J & \text{Configs} \\
\text{(iii)} & \sum_{a \in p \in P} x_p - \sum_{a \in q \in Q} y_q & \leq 0 & \forall a \in A & \text{Coupling} \\
\text{(iv)} & x_p & \in \{0,1\} & \forall p \in P & \text{Integ.} \\
\text{(v)} & y_q & \in \{0,1\} & \forall q \in Q & \text{Integ.}
\end{array}
$$

Theorem (B., Schlechte [2007]):

$$v_{LP}(PCP) = v_{LP}(ACP)$$

$$= v_{LP}\ (APP) = v_{LP}(PPP)$$

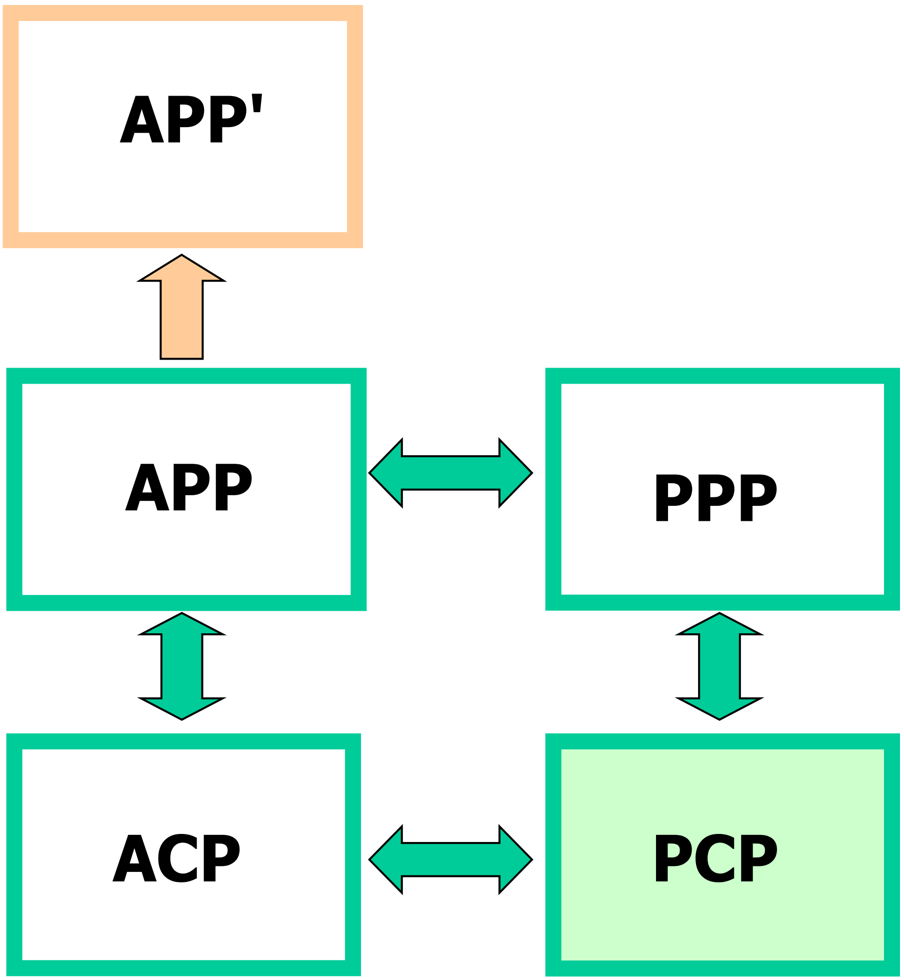
$$\leq v_{LP}(APP').$$

All LP-relaxations can be solved in polynomial time.

$$v_{IP}(PCP) = v_{IP}(ACP)$$

$$= v_{IP}\ (APP) = v_{IP}(PPP)$$

$$= v_{IP}(APP').$$



$$\text{(APP)} \quad \max \sum_{i\in I}\sum_{a\in A} c_a^i x_a^i$$

$$\begin{array}{llcll}
\text{(i)} & \displaystyle\sum_{a\in\delta_i^+(v)} x_a^i - \sum_{a\in\delta_i^-(v)} x_a^i & = & \beta_i(v) & \forall v\in V, i\in I & \text{Flow} \\
\text{(ii)} & \displaystyle\sum_{(a,i)\in k} x_a^i & \le & 1 & \forall k\in K & \text{Conflicts} \\
\text{(iii)} & x_a^i & \in & \{0,1\} & \forall a\in A, i\in I & \text{Integ.}
\end{array}$$

$$\text{(PCP)} \quad \max \sum_{i\in I}\sum_{p\in P_i}\sum_{a\in p} c_a^i x_p$$

$$\begin{array}{llcll}
\text{(i)} & \displaystyle\sum_{p\in P_i} x_p & \le & 1 & \forall i\in I & \text{Trains} \\
\text{(ii)} & \displaystyle\sum_{q\in Q_j} y_q & \le & 1 & \forall j\in J & \text{Configs} \\
\text{(iii)} & \displaystyle\sum_{a\in p\in P} x_p - \sum_{a\in q\in Q} y_q & \le & 0 & \forall a\in A & \text{Coupling} \\
\text{(iv)} & x_p & \in & \{0,1\} & \forall p\in P & \text{Integ.} \\
\text{(v)} & y_q & \in & \{0,1\} & \forall q\in Q & \text{Integ.}
\end{array}$$

$$
\begin{array}{llcll}
\text{(DUA)} & \min \sum_{i\in I}\gamma_i + \sum_{j\in J}\pi_j & & & \\
\text{(i)} & \gamma_i + \sum_{a\in p}\lambda_a \geq \sum_{a\in p} c_a^i & \forall p\in P_i, i\in I & \text{Paths} \\
\text{(ii)} & \pi_j - \sum_{a\in q}\lambda_a \geq 0 & \forall q\in Q_j, j\in J & \text{Configs} \\
\text{(iii)} & \gamma,\pi,\lambda \geq 0 & &
\end{array}
$$

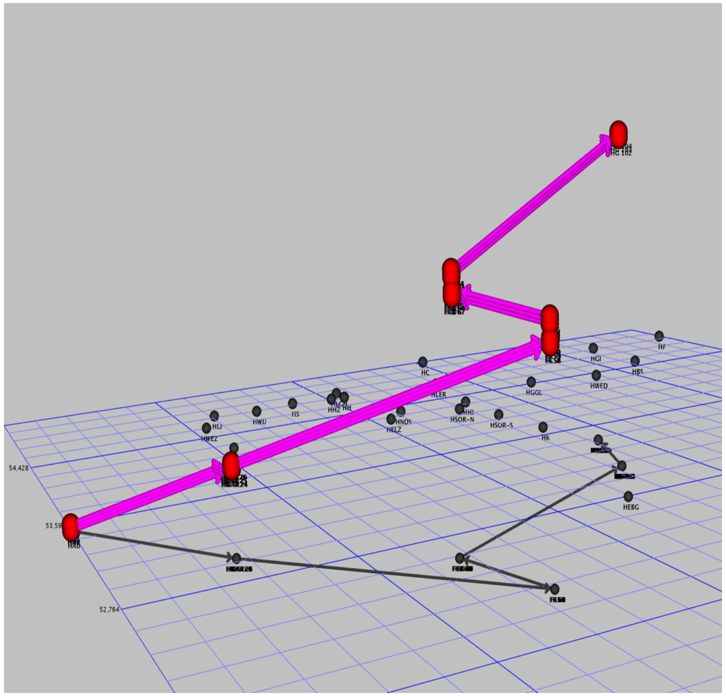
$$
\begin{array}{llcll}
\text{(PLP)} & \max \sum_{i\in I}\sum_{p\in P_i}\sum_{a\in p} c_a^i x_p & & & \\
\text{(i)} & \sum_{p\in P_i} x_p \leq 1 & \forall i\in I & \text{Trains} \\
\text{(ii)} & \sum_{q\in Q_j} y_q \leq 1 & \forall j\in J & \text{Configs} \\
\text{(iii)} & \sum_{a\in p\in P} x_p - \sum_{a\in q\in Q} y_q \leq 0 & \forall a\in A & \text{Coupling} \\
\text{(iv)} & x_p \geq 0 & \forall p\in P & \text{Integ.} \\
\text{(v)} & y_q \geq 0 & \forall q\in Q & \text{Integ.}
\end{array}
$$

$$
\begin{array}{llcll}
\text{(DUA)} & \min \sum_{i\in I}\gamma_i + \sum_{j\in J}\pi_j & & & \\
\text{(i)} & \gamma_i + \sum_{a\in p}\lambda_a \geq \sum_{a\in p} c_a^i & \forall p\in P_i, i\in I & \text{Paths} \\
\text{(ii)} & \pi_j - \sum_{a\in q}\lambda_a \geq 0 & \forall q\in Q_j, j\in J & \text{Configs} \\
\text{(iii)} & \gamma, \pi \geq 0 & &
\end{array}
$$

Proposition:

Route pricing = acyclic shortest path problem with arc weights

$$\bar{c}_a = -c_a + \lambda_a.$$

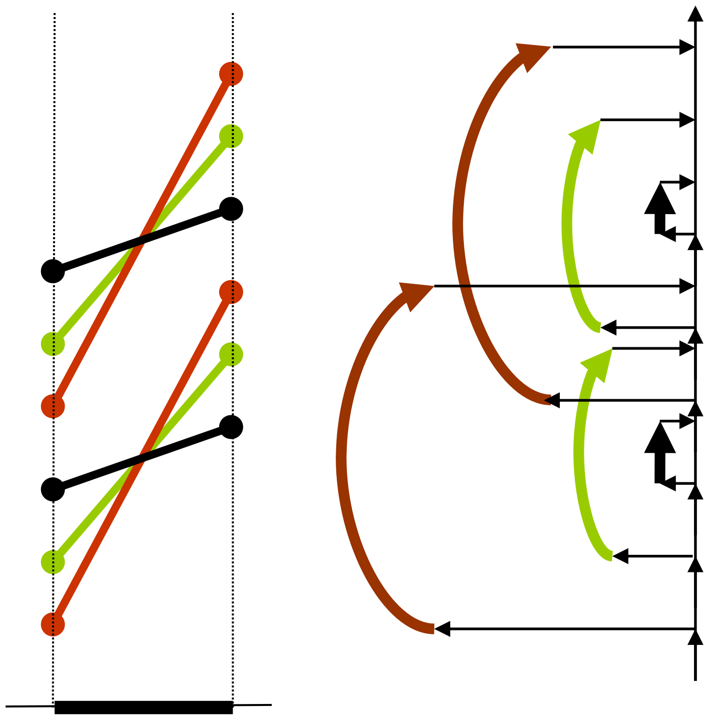


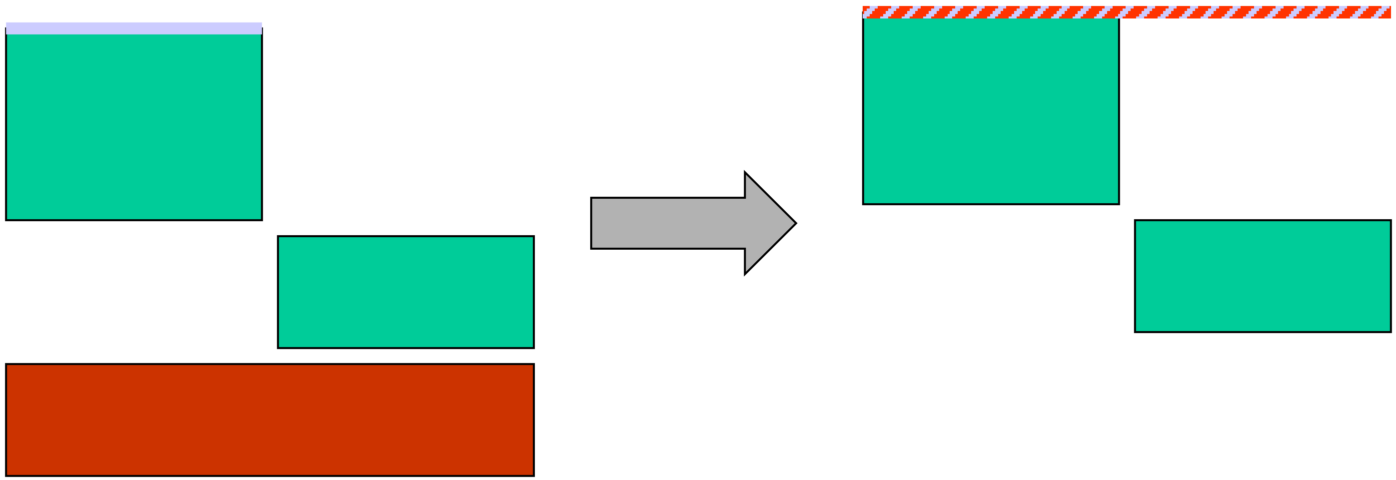
$$
\begin{array}{llcll}
\text{(DUA)} & \min \sum_{i \in I} \gamma_i + \sum_{j \in J} \pi_j & & & \\
\text{(i)} & \gamma_i + \sum_{a \in p} \lambda_a & \geq \sum_{a \in p} c_a^i & \forall p \in P_i, i \in I & \text{Paths} \\
\text{(ii)} & \pi_j - \sum_{a \in q} \lambda_a & \geq 0 & \forall q \in Q_j, j \in J & \text{Configs} \\
\text{(iii)} & \gamma, \pi & \geq 0 & &
\end{array}
$$

Proposition:

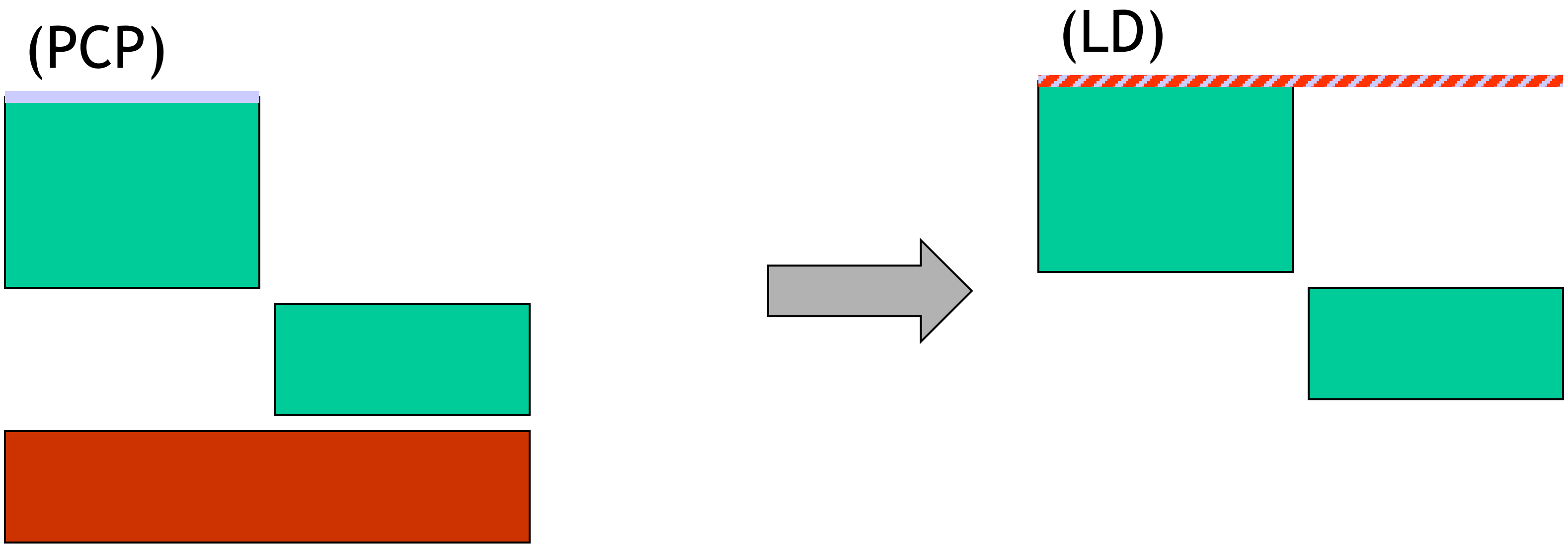
Config pricing = acyclic shortest path problem with arc weights

$$\bar{c}_a = -\lambda_a.$$





$$
\begin{array}{llllll}
\text{(PLP)} & \max \sum_{i \in I} \sum_{p \in P_i} \sum_{a \in p} c_a^i x_p & & & & \\
\text{(i)} & \sum_{p \in P_i} x_p & \le & 1 & \forall i \in I & \text{Trains} \\
\text{(ii)} & \sum_{q \in Q_j} y_q & \le & 1 & \forall j \in J & \text{Configs} \\
\text{(iii)} & \sum_{a \in p \in P} x_p - \sum_{a \in q \in Q} y_q & \le & 0 & \forall a \in A & \text{Coupling} \\
\text{(iv)} & x_p & \ge & 0 & \forall p \in P & \text{Integ.} \\
\text{(v)} & y_q & \ge & 0 & \forall q \in Q & \text{Integ.}
\end{array}
$$



$$(\mathrm{LD}) \quad \min_{\lambda \geq \mathbf{0}} \left[\max_{\substack{Ax=\mathbf{1},\\ x\in[0,1]^{|P|}}} (u^\top - \lambda^\top C)x + \max_{\substack{By=\mathbf{1},\\ y\in[0,1]^{|Q|}}} (\lambda^\top D)y \right]$$

Bundle Method

- ▷ Problem
- ▷ Algorithm
  - ▸ Subgradient
  - ▸ Cutting Plane Model
  - ▸ Update
- ▷ Quadratic Subproblem

$$f(\lambda) := \min_{x \in X} c^T x + \lambda^T (b - Ax)$$

$$\bar{f}_\mu(\lambda) = c^T x_\mu + \lambda^T (b - Ax_\mu)$$

$$\hat{f}_k(\lambda) := \min_{\mu \in J_k} \bar{f}_\mu(\lambda)$$

$$\lambda_{k+1} = \operatorname*{argmax}_\lambda \hat{f}_k(\lambda) - \frac{u_k}{2} \left\| \lambda - \hat{\lambda}_k \right\|^2$$

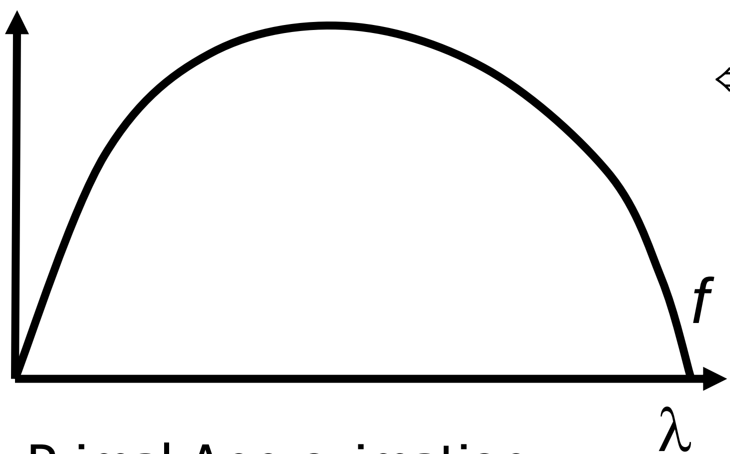
$$\max \hat{f}_k(\lambda) - \frac{u_k}{2} \left\| \lambda - \hat{\lambda}_k \right\|^2 \Leftrightarrow \max \quad v - \frac{u_k}{2} \left\| \lambda - \hat{\lambda}^k \right\|^2$$

$$\text{s.t.} \quad v \le \bar{f}_\mu(\lambda), \text{ for all } \mu \in J_k$$

$$\Leftrightarrow \max \quad \sum_{\mu \in J_k} \alpha_\mu \bar{f}_\mu(\hat{\lambda}) - \frac{1}{2u_k} \left\| \sum_{\mu \in J_k} \alpha_\mu (b - Ax_\mu) \right\|^2$$

$$\text{s.t.} \quad \sum_{\mu \in J_k} \alpha_\mu = 1$$

$$0 \le \alpha_\mu \le 1, \quad \text{for all } \mu \in J_k$$



- ▷ Primal Approximation
- ▷ Inexact Bundle Method

$$\left\| b - A\tilde{x}_k \right\| \to 0 \ (k \to \infty) \qquad \tilde{x}_{k+1} = \sum_{\mu \in J_k} \alpha_\mu x_\mu$$

- Problem
- Algorithm
  - Subgradient
  - Cutting Plane Model
  - Update
- Quadratic Subproblem

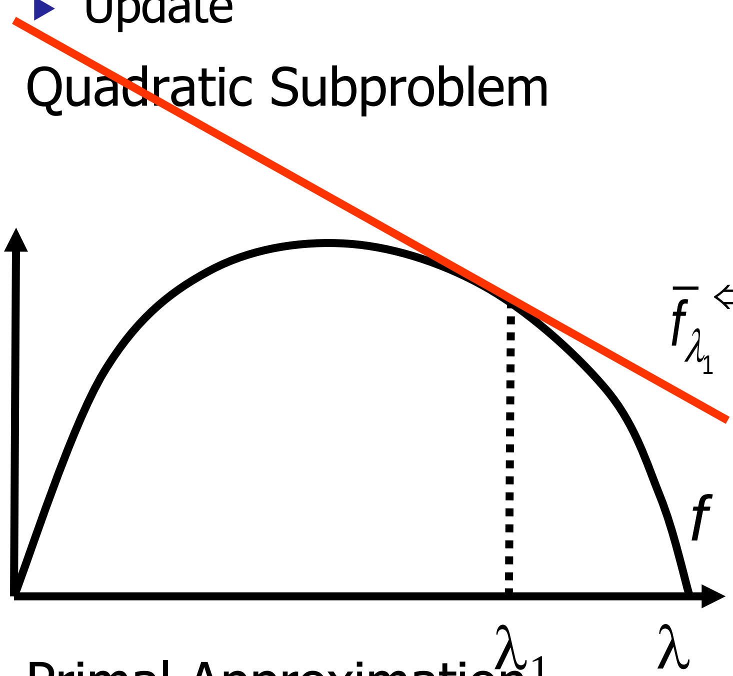
$$f(\lambda) := \min_{x \in X} c^{\mathrm{T}} x + \lambda^{\mathrm{T}}(b - Ax)$$

$$\bar{f}_\mu(\lambda) = c^{\mathrm{T}} x_\mu + \lambda^{\mathrm{T}}(b - Ax_\mu)$$

$$\hat{f}_k(\lambda) := \min_{\mu \in J_k} \bar{f}_\mu(\lambda)$$

$$\lambda_{k+1} = \operatorname*{argmax}_\lambda \hat{f}_k(\lambda) - \frac{u_k}{2}\left\|\lambda - \hat{\lambda}_k\right\|^2$$

$$\max \hat{f}_k(\lambda) - \frac{u_k}{2}\left\|\lambda - \hat{\lambda}_k\right\|^2 \Leftrightarrow \max \quad v - \frac{u_k}{2}\left\|\lambda - \hat{\lambda}^k\right\|^2$$
$$\text{s.t.} \quad v \le \bar{f}_\mu(\lambda), \text{ for all } \mu \in J_k$$



$$\Leftrightarrow \max \quad \sum_{\mu \in J_k} \alpha_\mu \bar{f}_\mu(\hat{\lambda}) - \frac{1}{2u_k}\left\|\sum_{\mu \in J_k} \alpha_\mu (b - Ax_\mu)\right\|^2$$
$$\text{s.t.} \quad \sum_{\mu \in J_k} \alpha_\mu = 1$$
$$0 \le \alpha_\mu \le 1, \quad \text{for all } \mu \in J_k$$

- Primal Approximation
- Inexact Bundle Method

$$\left\|b - A\tilde{x}_k\right\| \to 0 \ (k \to \infty) \qquad \tilde{x}_{k+1} = \sum_{\mu \in J_k} \alpha_\mu x_\mu$$

Bundle Method

(Kiwiel [1990], Helmberg [2000])

- ▷ Problem
- ▷ Algorithm
  - ▸ Subgradient
  - ▸ Cutting Plane Model
  - ▸ Update
- ▷ Quadratic Subproblem

$$f(\lambda) := \min_{x \in X} c^{\mathrm{T}} x + \lambda^{\mathrm{T}} (b - Ax)$$

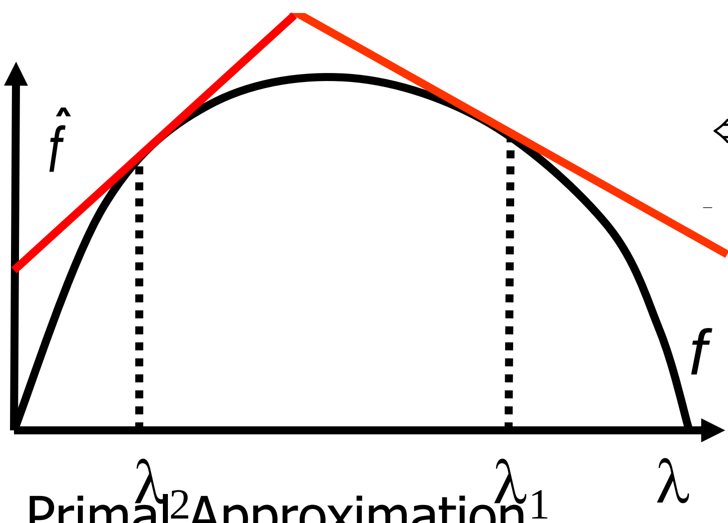
$$\bar{f}_\mu(\lambda) = c^{\mathrm{T}} x_\mu + \lambda^{\mathrm{T}} (b - Ax_\mu)$$

$$\hat{f}_k(\lambda) := \min_{\mu \in J_k} \bar{f}_\mu(\lambda)$$

$$\lambda_{k+1} = \operatorname*{argmax}_{\lambda} \hat{f}_k(\lambda) - \frac{u_k}{2} \left\| \lambda - \hat{\lambda}_k \right\|^2$$

$$\max \hat{f}_k(\lambda) - \frac{u_k}{2} \left\| \lambda - \hat{\lambda}_k \right\|^2 \Leftrightarrow \max \quad v - \frac{u_k}{2} \left\| \lambda - \hat{\lambda}^k \right\|^2$$

$$\text{s.t.} \quad v \le \bar{f}_\mu(\lambda), \text{ for all } \mu \in J_k$$



$$\Leftrightarrow \max \quad \sum_{\mu \in J_k} \alpha_\mu \bar{f}_\mu(\hat{\lambda}) - \frac{1}{2u_k} \left\| \sum_{\mu \in J_k} \alpha_\mu (b - Ax_\mu) \right\|^2$$

$$\text{s.t.} \quad \sum_{\mu \in J_k} \alpha_\mu = 1$$

$$0 \le \alpha_\mu \le 1, \qquad \text{for all } \mu \in J_k$$

- ▷ Primal Approximation
- ▷ Inexact Bundle Method

$$\left\| b - A\tilde{x}_k \right\| \to 0 \ (k \to \infty) \qquad \tilde{x}_{k+1} = \sum_{\mu \in J_k} \alpha_\mu x_\mu$$

- Problem
- Algorithm
  - Subgradient
  - Cutting Plane Model
  - Update
- Quadratic Subproblem

$$f(\lambda) := \min_{x \in X} c^{\mathrm{T}} x + \lambda^{\mathrm{T}}(b - Ax)$$

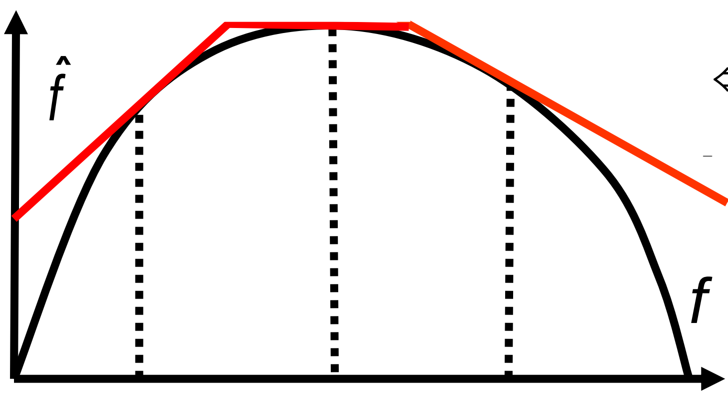
$$\bar{f}_\mu(\lambda) = c^{\mathrm{T}} x_\mu + \lambda^{\mathrm{T}}(b - Ax_\mu)$$

$$\hat{f}_k(\lambda) := \min_{\mu \in J_k} \bar{f}_\mu(\lambda)$$

$$\lambda_{k+1} = \operatorname*{argmax}_{\lambda} \hat{f}_k(\lambda) - \frac{u_k}{2}\left\|\lambda - \hat{\lambda}_k\right\|^2$$

$$\max \hat{f}_k(\lambda) - \frac{u_k}{2}\left\|\lambda - \hat{\lambda}_k\right\|^2 \Leftrightarrow \max \quad v - \frac{u_k}{2}\left\|\lambda - \hat{\lambda}^k\right\|^2$$
$$\text{s.t.} \quad v \le \bar{f}_\mu(\lambda), \text{ for all } \mu \in J_k$$

$$\Leftrightarrow \max \quad \sum_{\mu \in J_k} \alpha_\mu \bar{f}_\mu(\hat{\lambda}) - \frac{1}{2u_k}\left\|\sum_{\mu \in J_k} \alpha_\mu (b - Ax_\mu)\right\|^2$$
$$\text{s.t.} \quad \sum_{\mu \in J_k} \alpha_\mu = 1$$
$$0 \le \alpha_\mu \le 1, \qquad \text{for all } \mu \in J_k$$



- Primal Approximation
- Inexact Bundle Method

$$\left\|b - A\tilde{x}_k\right\| \to 0 \ (k \to \infty) \qquad \tilde{x}_{k+1} = \sum_{\mu \in J_k} \alpha_\mu x_\mu$$

- ▷ Problem

$$f(\lambda) := \min_{x \in X} c^T x + \lambda^T (b - Ax)$$

- ▷ Algorithm
  - ▸ Subgradient

$$\bar{f}_\mu(\lambda) = c^T x_\mu + \lambda^T (b - A x_\mu)$$

  - ▸ Cutting Plane Model

$$\hat{f}_k(\lambda) := \min_{\mu \in J_k} \bar{f}_\mu(\lambda)$$

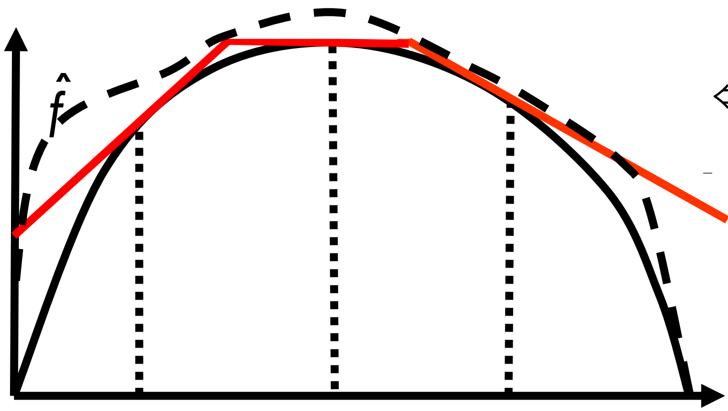
  - ▸ Update

$$\lambda_{k+1} = \operatorname*{argmax}_{\lambda} \hat{f}_k(\lambda) - \frac{u_k}{2} \left\| \lambda - \hat{\lambda}_k \right\|^2$$

- ▷ Quadratic Subproblem

$$\max \hat{f}_k(\lambda) - \frac{u_k}{2} \left\| \lambda - \hat{\lambda}_k \right\|^2 \Leftrightarrow \begin{array}{ll} \max & v - \frac{u_k}{2} \left\| \lambda - \hat{\lambda}^k \right\|^2 \\ \text{s.t.} & v \le \bar{f}_\mu(\lambda), \text{ for all } \mu \in J_k \end{array}$$

$$\Leftrightarrow \begin{array}{ll} \max & \sum_{\mu \in J_k} \alpha_\mu \bar{f}_\mu(\hat{\lambda}) - \frac{1}{2u_k} \left\| \sum_{\mu \in J_k} \alpha_\mu (b - A x_\mu) \right\|^2 \\ \text{s.t.} & \sum_{\mu \in J_k} \alpha_\mu = 1 \\ & 0 \le \alpha_\mu \le 1, \quad \text{for all } \mu \in J_k \end{array}$$



- ▷ Primal Approximation

$$\left\| b - A\tilde{x}_k \right\| \to 0 \ (k \to \infty) \qquad \tilde{x}_{k+1} = \sum_{\mu \in J_k} \alpha_\mu x_\mu$$

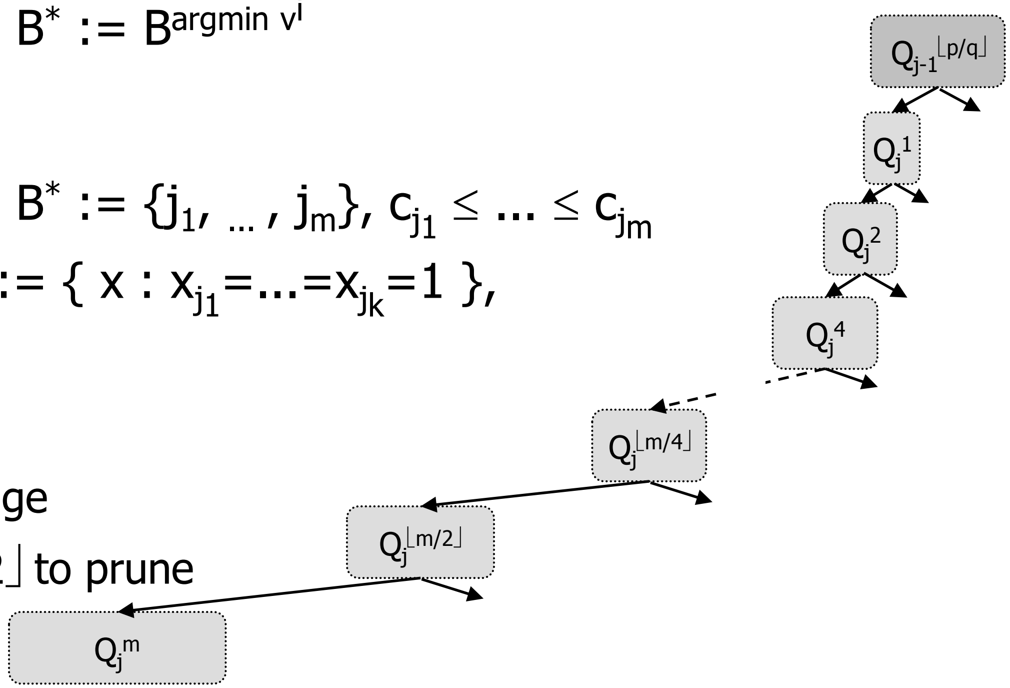
- ▷ Inexact Bundle Method

Perturbation Branching

- Sequence of perturbed IP objectives $c_j^{i+1} := c_j^i - \alpha(x_j^i)^2, \ \forall j, i=1,2,\ldots$
- Fixing candidates in iteration i $B^i := \{ j : x_j^i \geq 1 - \varepsilon \}$
- Potential function in iteration i $v^i := c^T x^i - w|B^i|$
- Go on while not integer and potential decreases, else
  - Perturb for k_{max} additional iterations, if still not successful
    - Fix a single variable and reset objective every k_s iterations
- Set of fixed variables (many) $B^* := B^{\text{argmin } v^i}$

Binary Search Branching

- Set of fixed variables (many) $B^* := \{j_1, \ldots, j_m\}, c_{j_1} \leq \ldots \leq c_{j_m}$
- Sets Q_j^k at pertubation branch j $Q_j^k := \{ x : x_{j_1} = \ldots = x_{j_k} = 1 \},$ $k=0,\ldots,m$
- Branch on Q_j^m
  - Repeat perturbation branching to plunge
  - Backtrack to $Q_j^{\lfloor m/2 \rfloor}$ and set $m := \lfloor m/2 \rfloor$ to prune



(PRICE (x)) $\exists\, \overline{p} \in \mathcal{P}_i : \quad \gamma_i < \sum_{a\in\overline{p}} (p_a - \lambda_a)$

$$\eta_i := \max_{p\in P_i} \sum_{a\in p} (p_a - \lambda_a) - \gamma_i,\ \forall i \in I \Longrightarrow \eta_i + \gamma_i \geq \sum_{a\in p} (p_a - \lambda_a)\ \forall i \in I, p \in \mathcal{P}_i$$

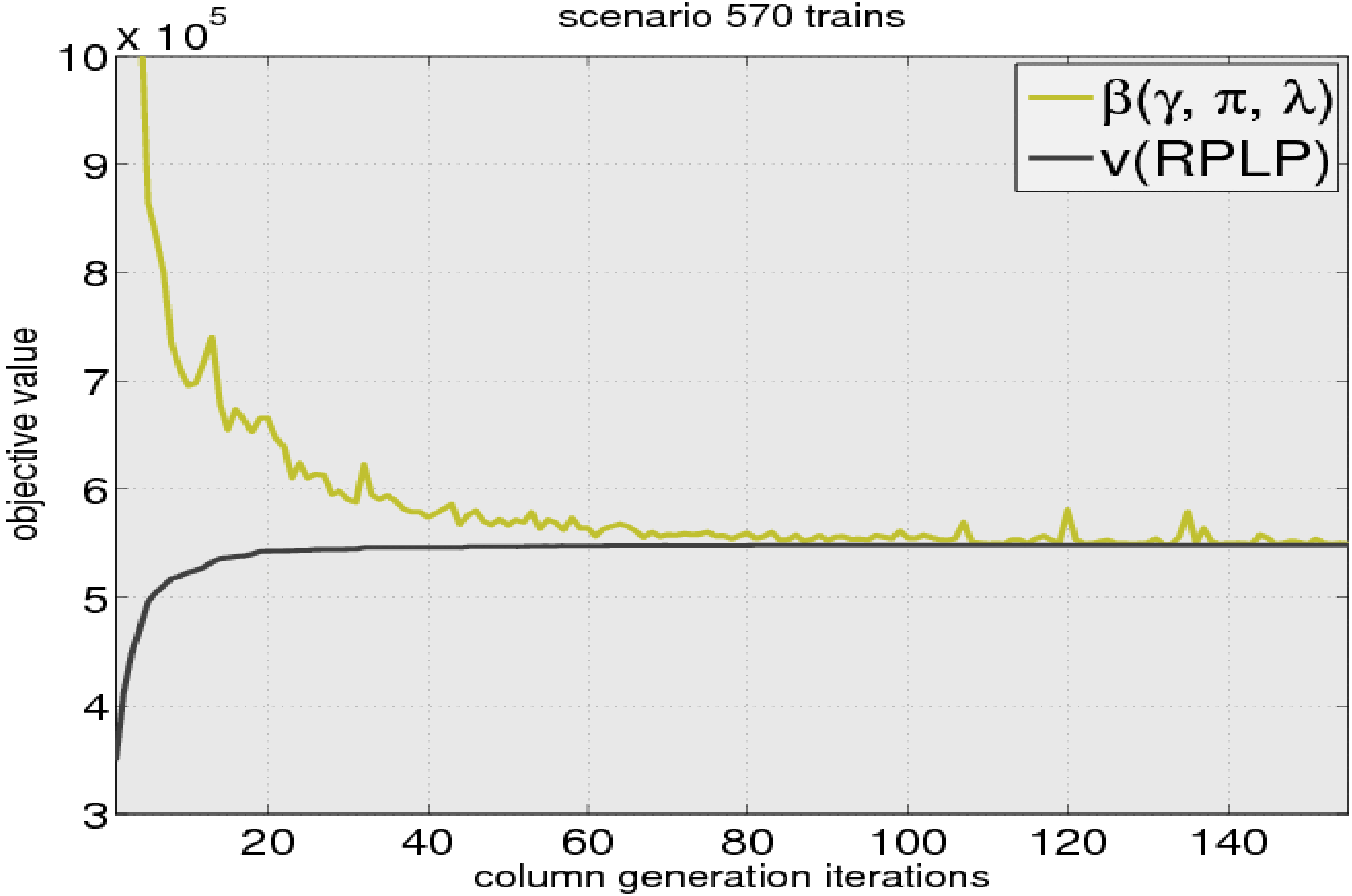
(PRICE (y)) $\exists\, \overline{q} \in Q_j : \quad \pi_j < \sum_{a\in\overline{q}} \lambda_a$

$$\theta_j := \max_{\overline{q}\in Q_j} \sum_{a\in\overline{q}} \lambda_a - \pi_j,\ \forall j \in J \Longrightarrow \theta_j + \pi_j \geq \sum_{a\in q} \lambda_a\ \forall j \in J, q \in \mathcal{Q}_j$$

$(\max\{\eta+\gamma, 0\}, \max\{\theta+\pi, 0\}, \lambda)$ is feasible for (DLP)

$$\beta(\gamma,\pi,\lambda) := \sum_{i\in I} \max\{\gamma_i+\eta_i, 0\} + \sum_{j\in J} \max\{\pi_j+\theta_j, 0\}$$

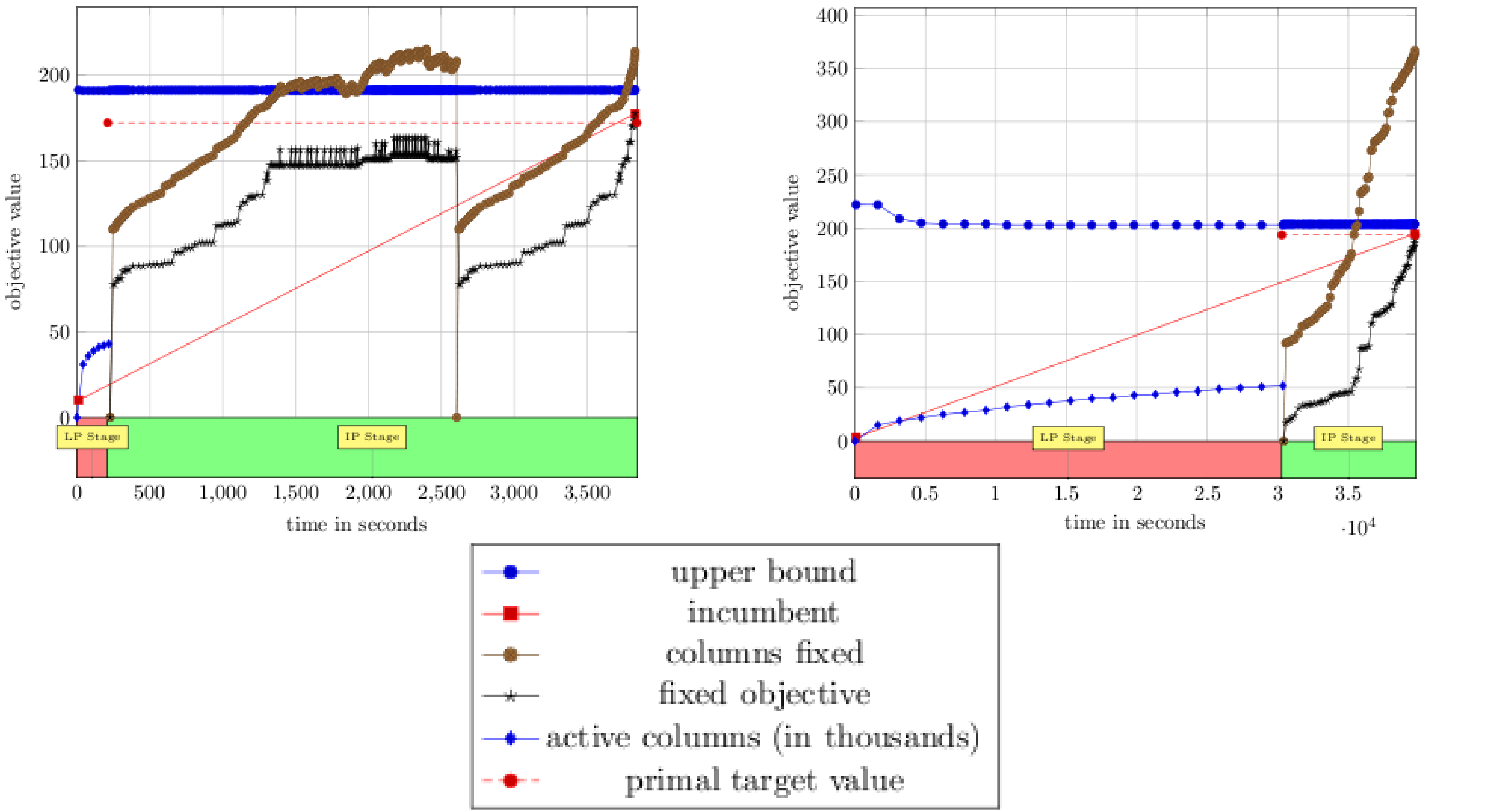
Lemma (BS [2007]): $v_{LP}(PCP) \leq \beta(\gamma,\pi,\lambda)$


scenario 570 trains
x 10^5
10
9
8
7
6
5
4
3
objective value
20
40
60
80
100
120
140
column generation iterations
β(γ, π, λ)
v(RPLP)

▷ HaKaFu, req32, 1140 requests, 30 mins time windows

TS-OPT run, model PCP, PCP-24H-NS-BUNDLE-BNB-100401-17:22:40

TS-OPT run, model PCP, PCP-24H-NS-BUNDLE-BNB-TW-30-100331-15:46:57



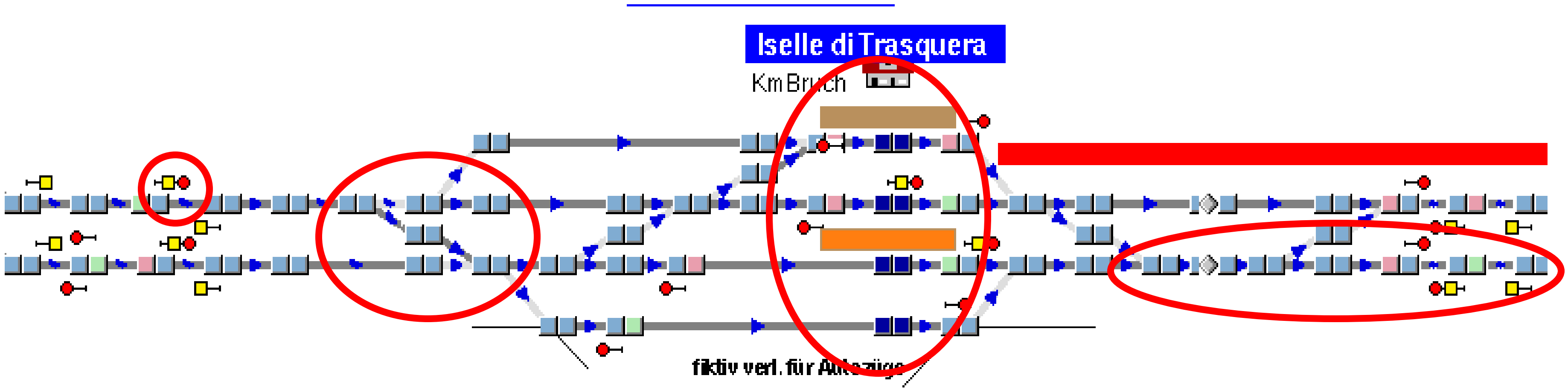
Track Allocation and Train Timetabling

Article	*Stations*	*Tracks*	*Trains*	*Modell/Approach*
Szpigel [1973]	6	5	10	Packing/Enumeration
Brännlund et al. [1998]	17	16	26	Packing/ Lagrange, BAB
Caprara et al. [2002]	74 (17)	73 (16)	54 (221)	Packing/ Lagrange, BAB
B. & Schlechte [2007]	37	120	570	Config/PAB
Caprara et al. [2007]	102 (16)	103 (17)	16 (221)	Packing/PAB
Fischer et al. [2008]	656 (104)	1210 (193)	117 (251)	Packing/Bundle, IP Rounding
Lusby et al. [2008]	???	524	66 (31)	Packing/BAP
B. & Schlechte [2010]	37	120	>1.000	Config/Rapid Branching

- BAB: Branch-and-Bound
- PAB: Price-and-Branch
- BAP: Branch-and-Price

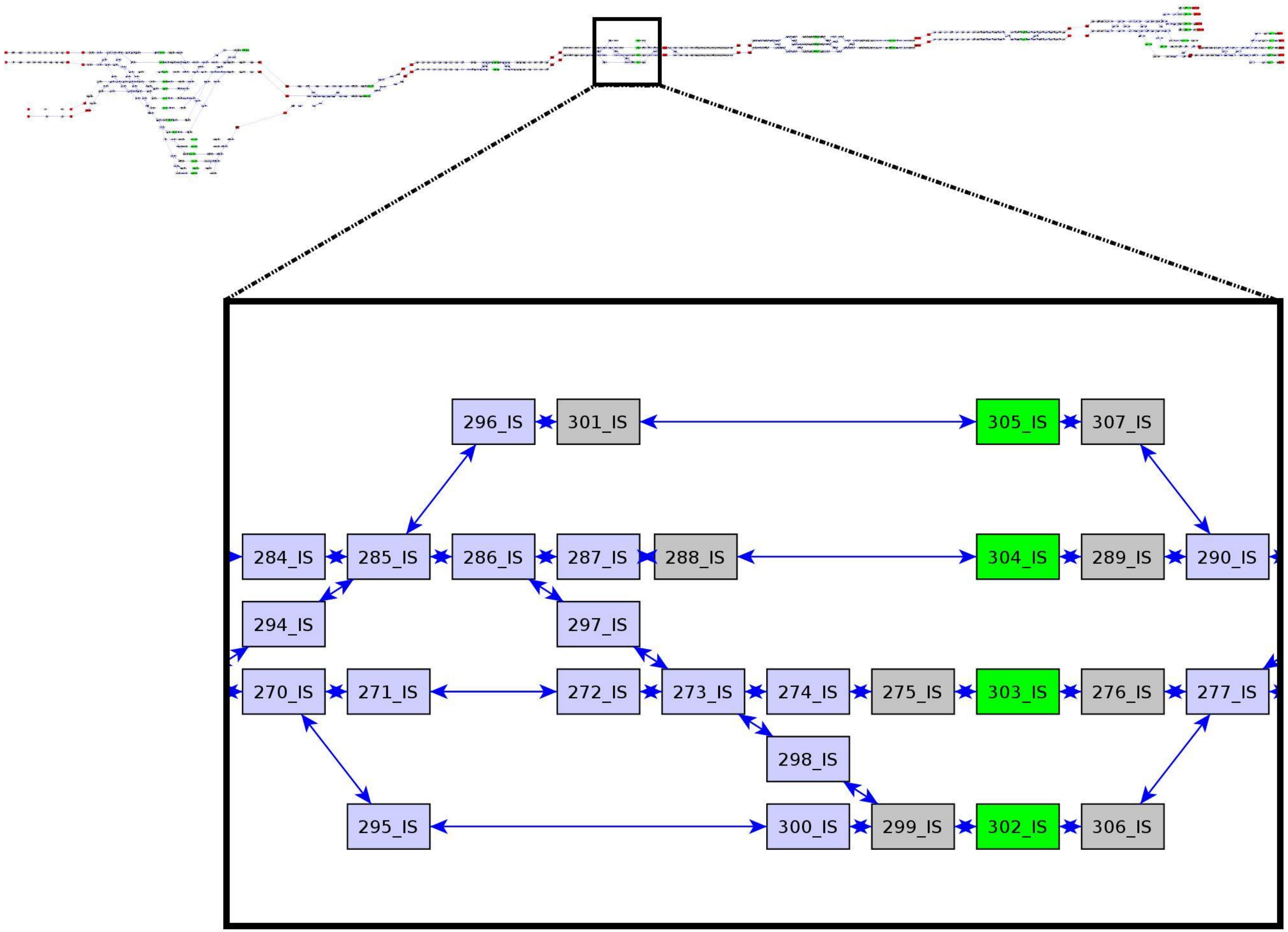
Discretization and Scheduling

▷ Detailed railway infrastucture data given by simulation programs (Open Track)

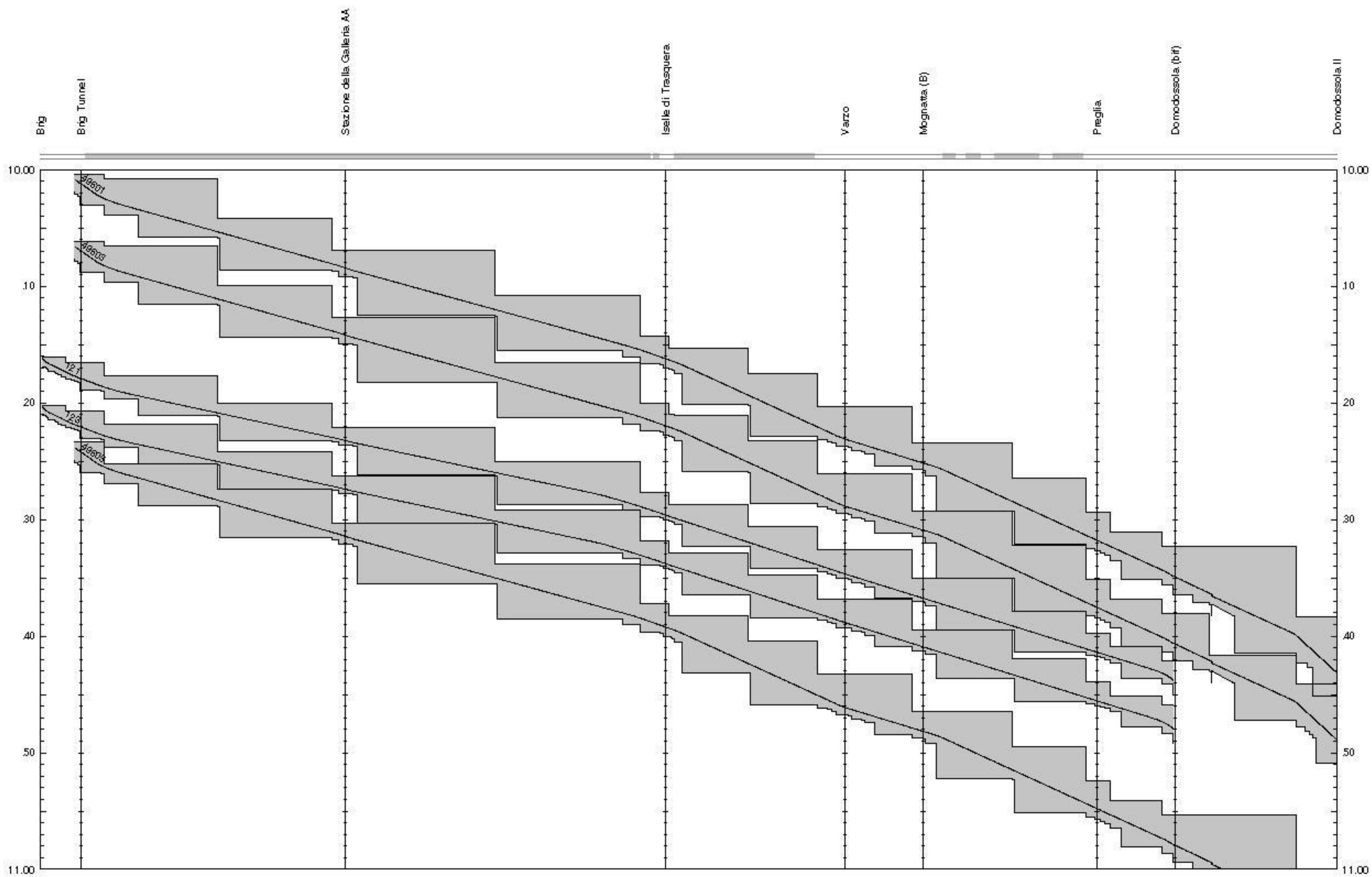


- ▷ Signals
- ▷ Switches
- ▷ Tracks (with max. speed, acceleration, gradient)
- ▷ Stations and Platforms

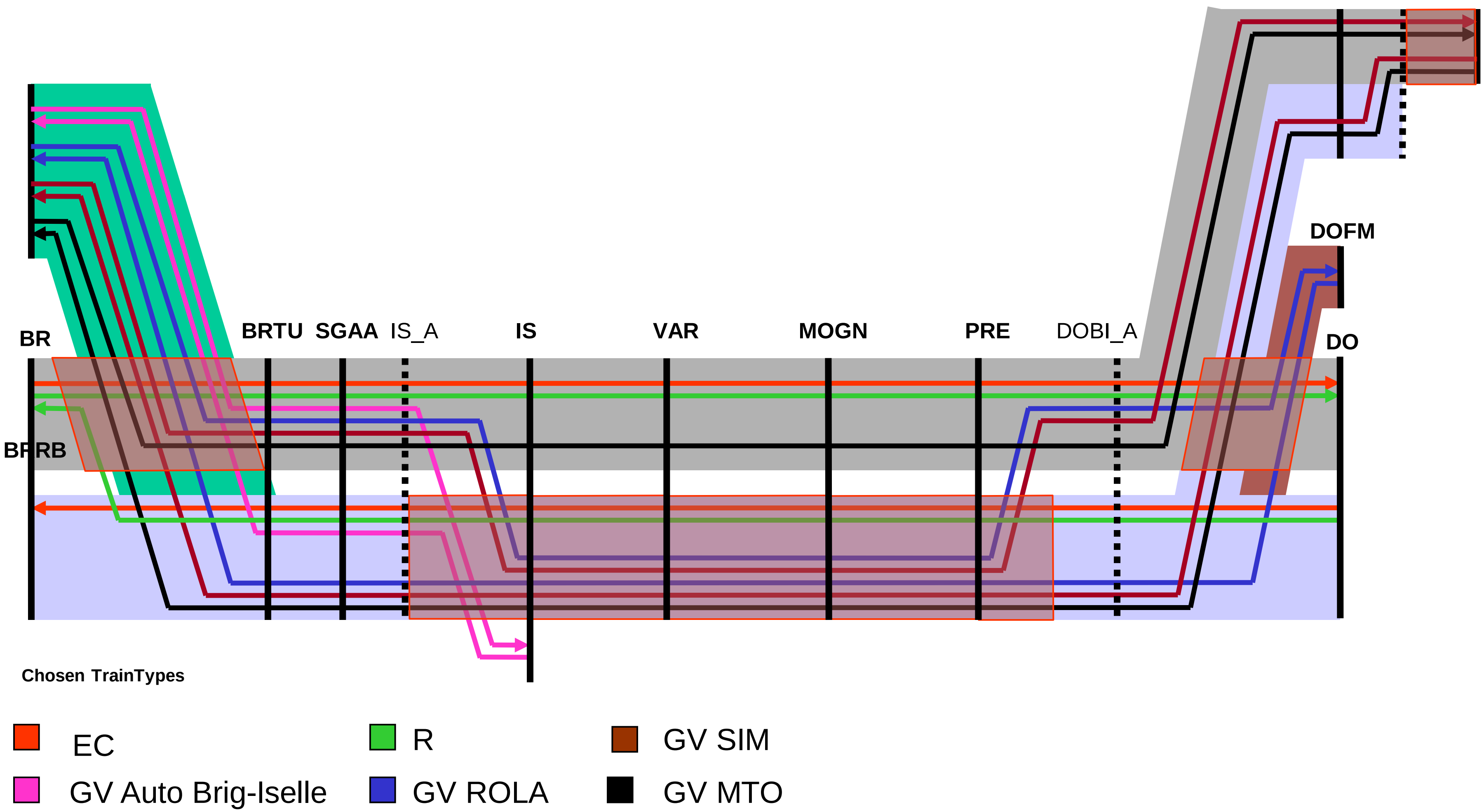
- Simplon micrograph: 1154 nodes and 1831 arcs, 223 signals etc.



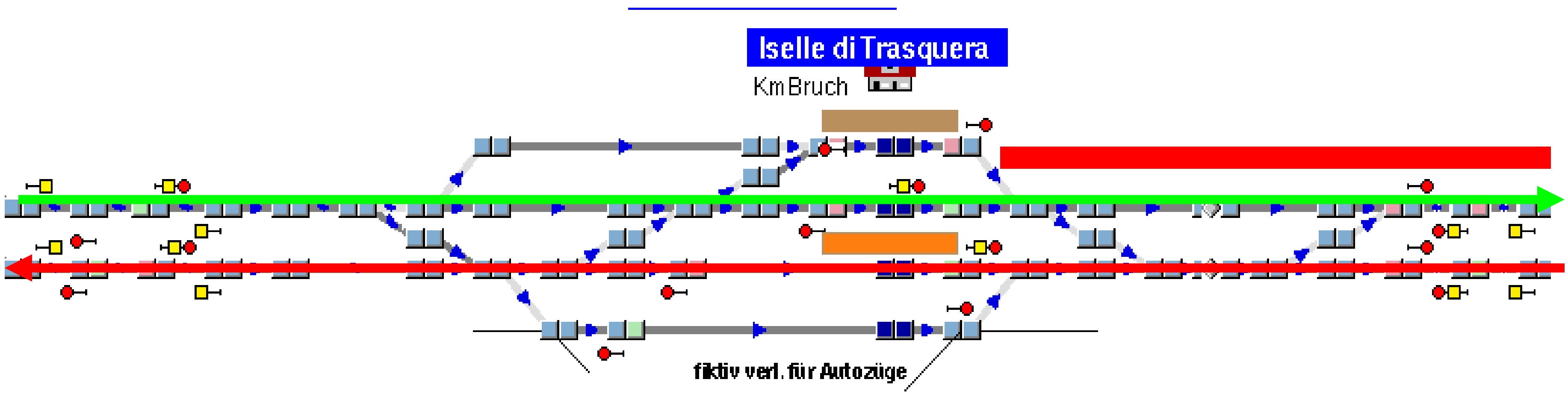
- Simulation tools provide exact running and blocking times
- Basis for calculation of minimal headway times



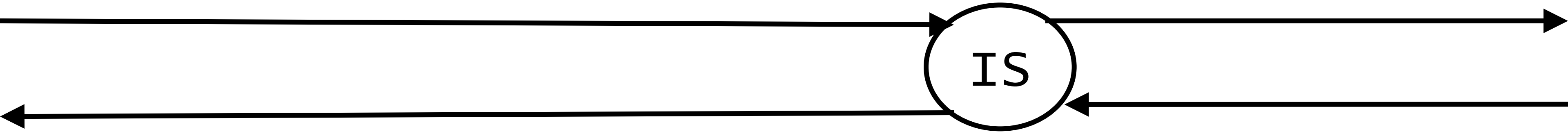
- Simulation of all possible routes with appropiate train types



- Generation of artifical nodes – „pseudo“ stations

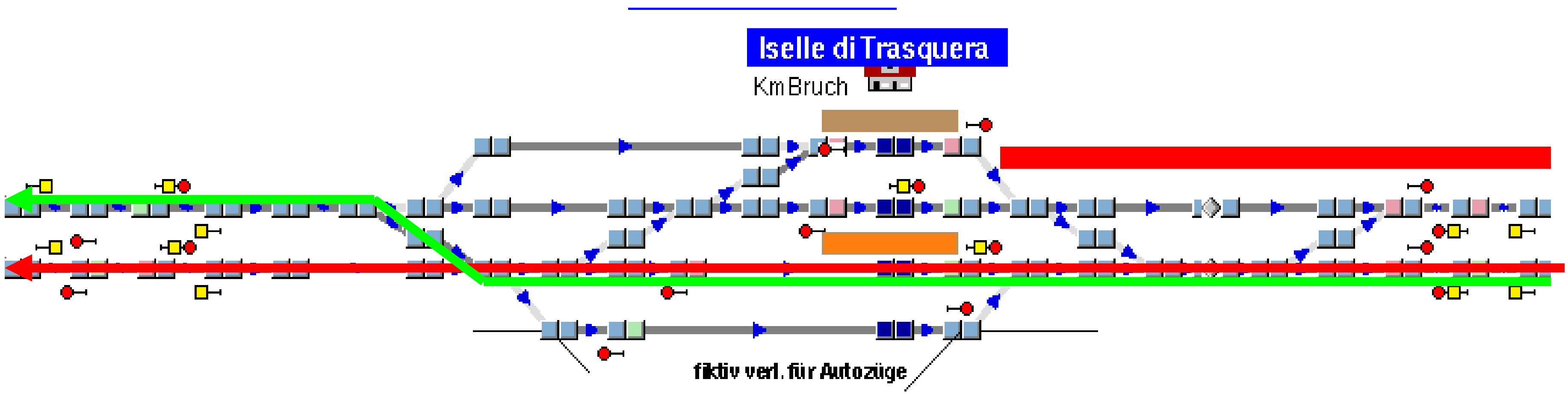


- No interactions between train routes

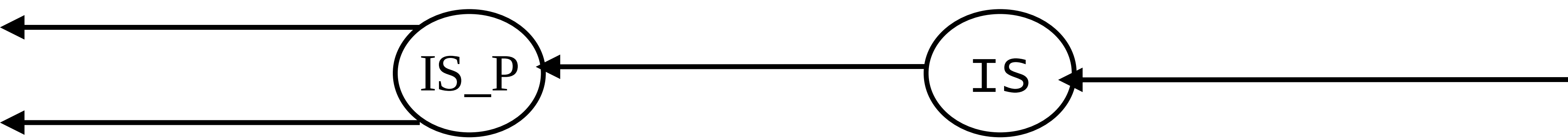


- Macro network definition is based on set of train routes

- Generation of artifical nodes – „pseudo“ stations

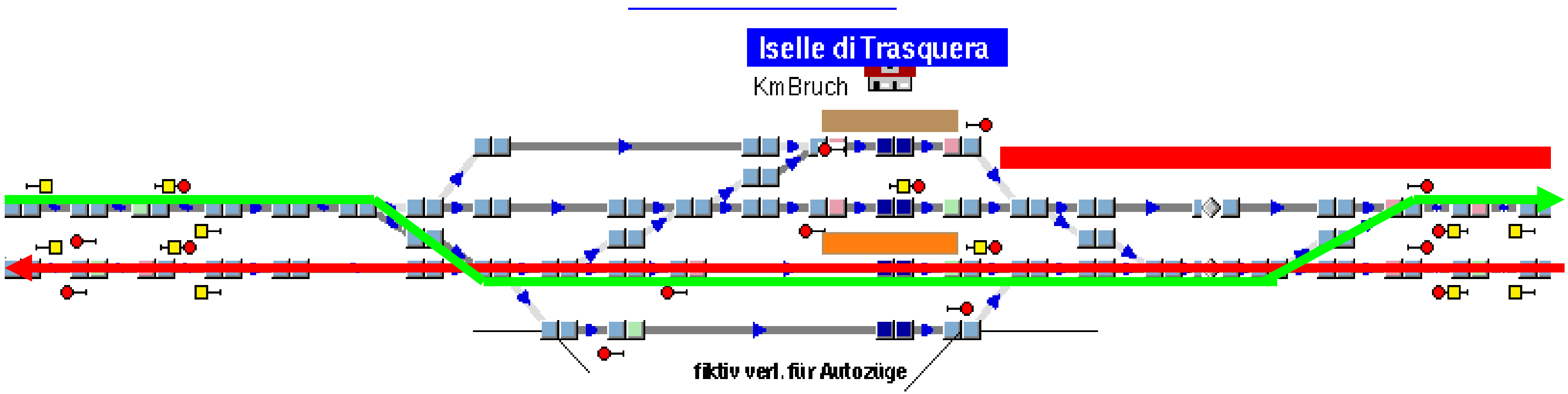


- Diverging of train routes

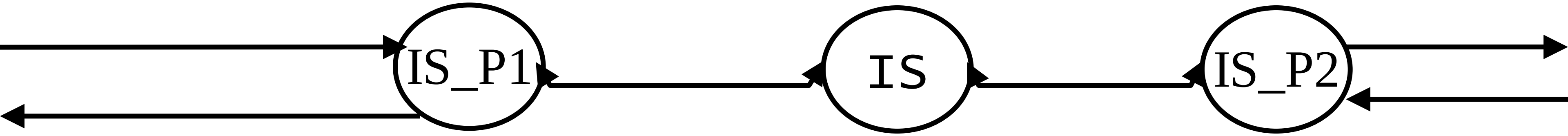


- The same holds for converging routes

▷ Generation of artifical nodes – pseudo stations



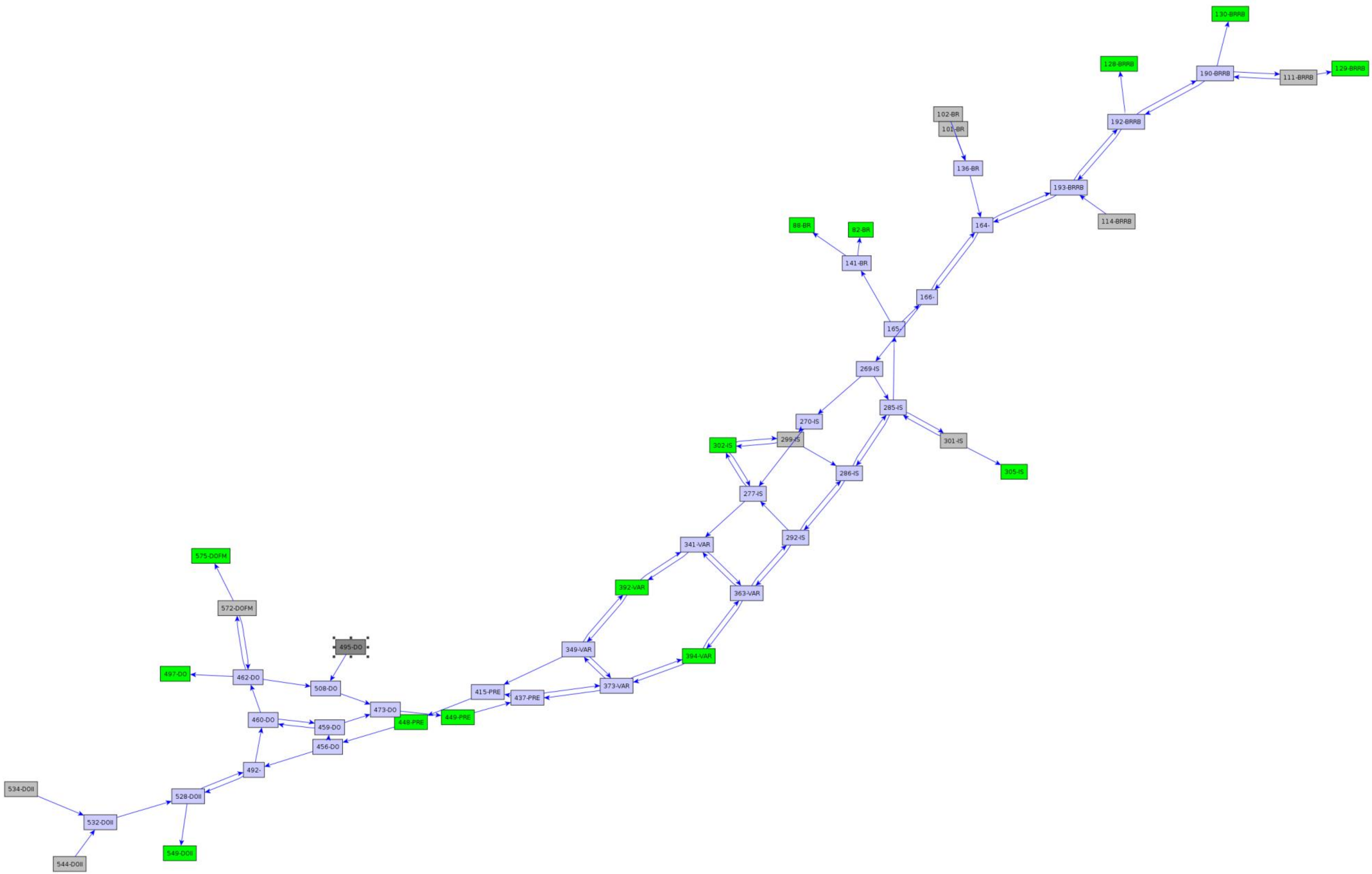
▷ crossing of train routes



▷ Two pseudo stations were generated

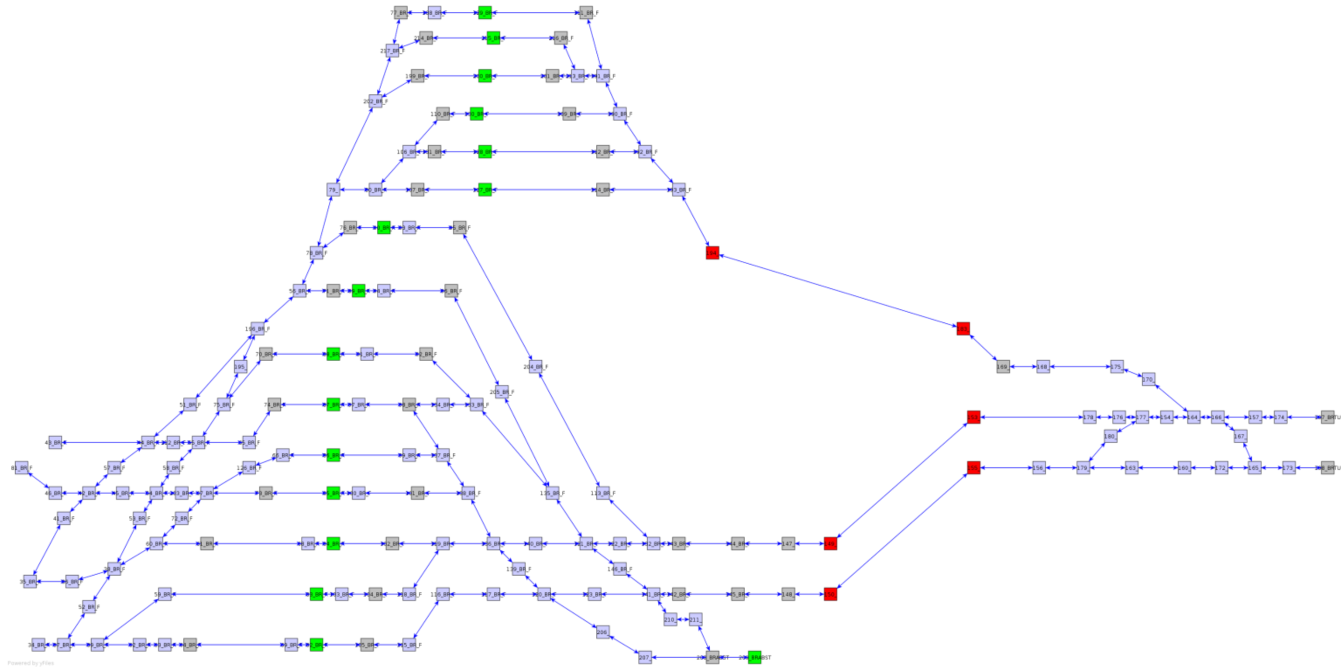
Reduced Macrograph

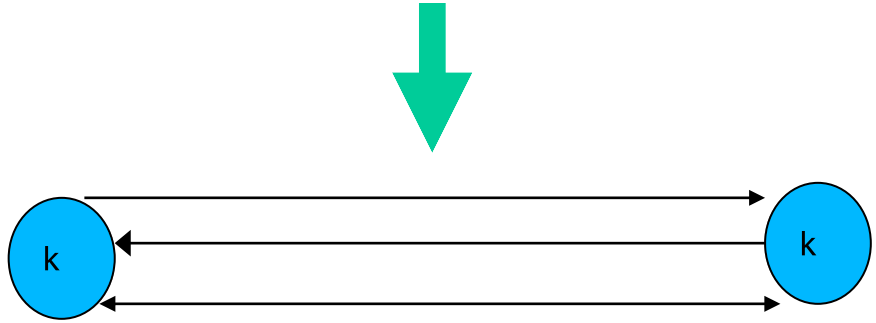
(53 nodes and 87 track arcs for 28 train routes)



Station Aggregation

- Frequently many macroscopic station nodes are in the area of big stations
- Further aggregation is needed





k =

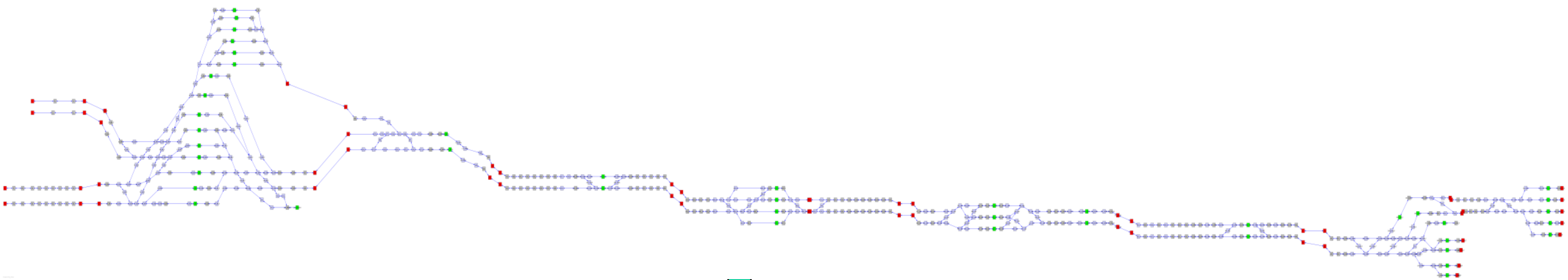
EC	2
R	4
GV Auto	2
GV Rola	2
GV SIM	4
GV MTO	6

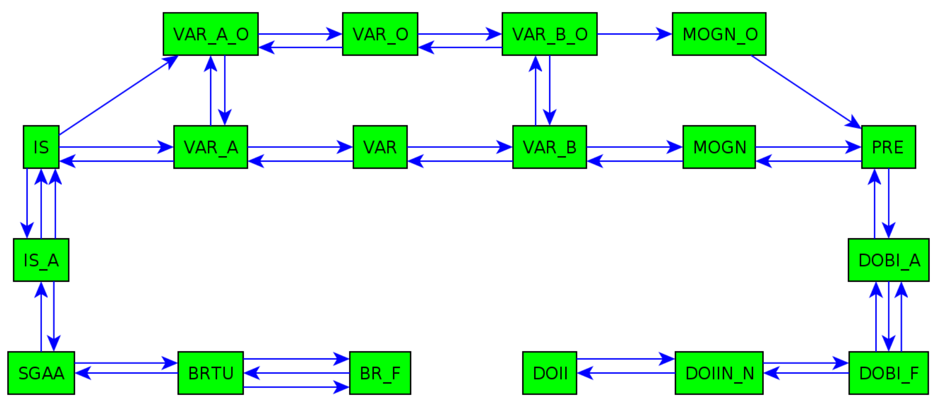
Micro-Macro Transformation

▷ Planned times in macro network are possible in micro network

▷ Valid headways lead to valid block occupations (no conflicts)

$\Rightarrow$ feasible macro timetable can be transformed to feasible micro timetable





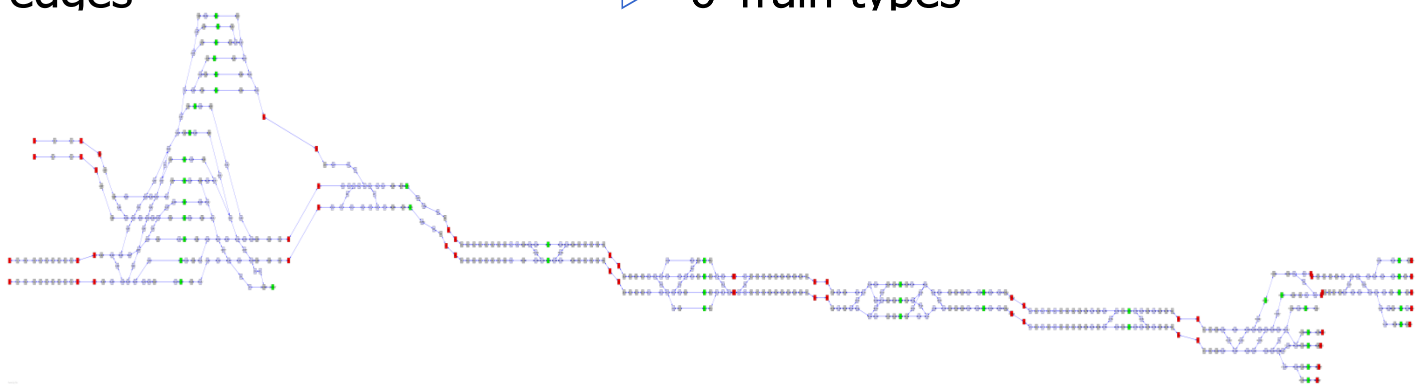
Micro-Macro-Transformation: Simplon Case

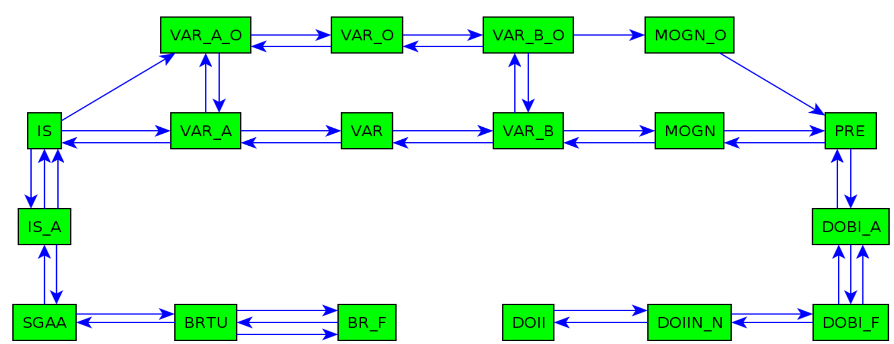
Micro

- 12 stations
- 1154 OpenTrack nodes
- 1831 OpenTrack edges
- 223 signals
- 8 track junctions
- 100 switches
- 6 train types
- 28 “routes“
- 230 ”block segments“

Macro

- 18 macro nodes
- 40 tracks
- 6 Train types



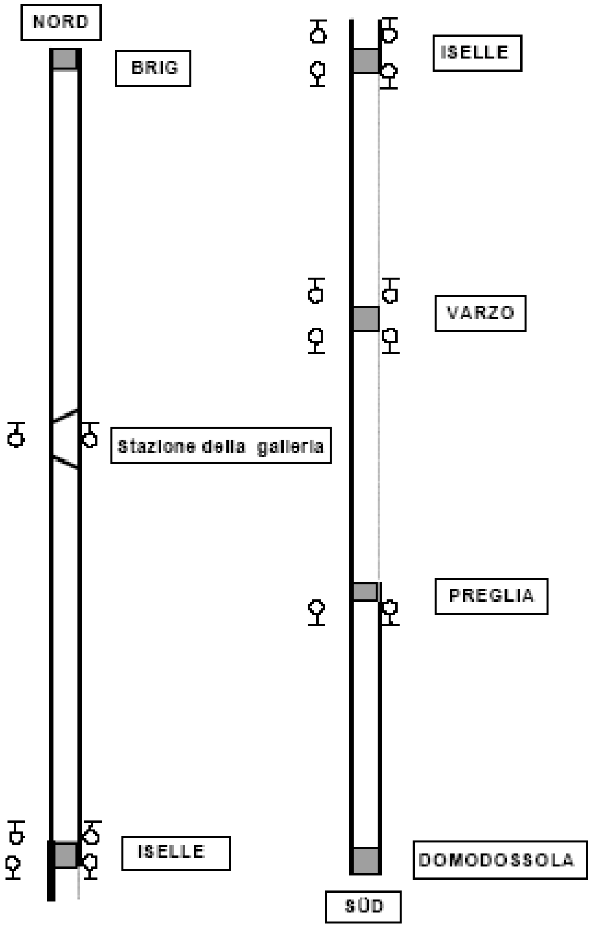


Cumulative Rounding Procedure

- ▷ Compute macroscopic running time with specific rounding procedure
- ▷ Consider again routes of trains (represented by standard trains)
- ▷ Example with $\Delta = 6$

Station	Dep/Pass	Rounded	Buffer
A	0	0	0
B	11	12 (2)	1
C	20	24 (4)	4
D	29	30 (5)	1

- ▷ **Theorem:** If micro-running time $d \geq \Delta$ for all tracks of the current train route, the cumulative rounding error (buffer) is always in $[0, \Delta)$.



Source: Wikipedia

Slalom route

- ROLA trains traverse the tunnel on the “wrong“ side

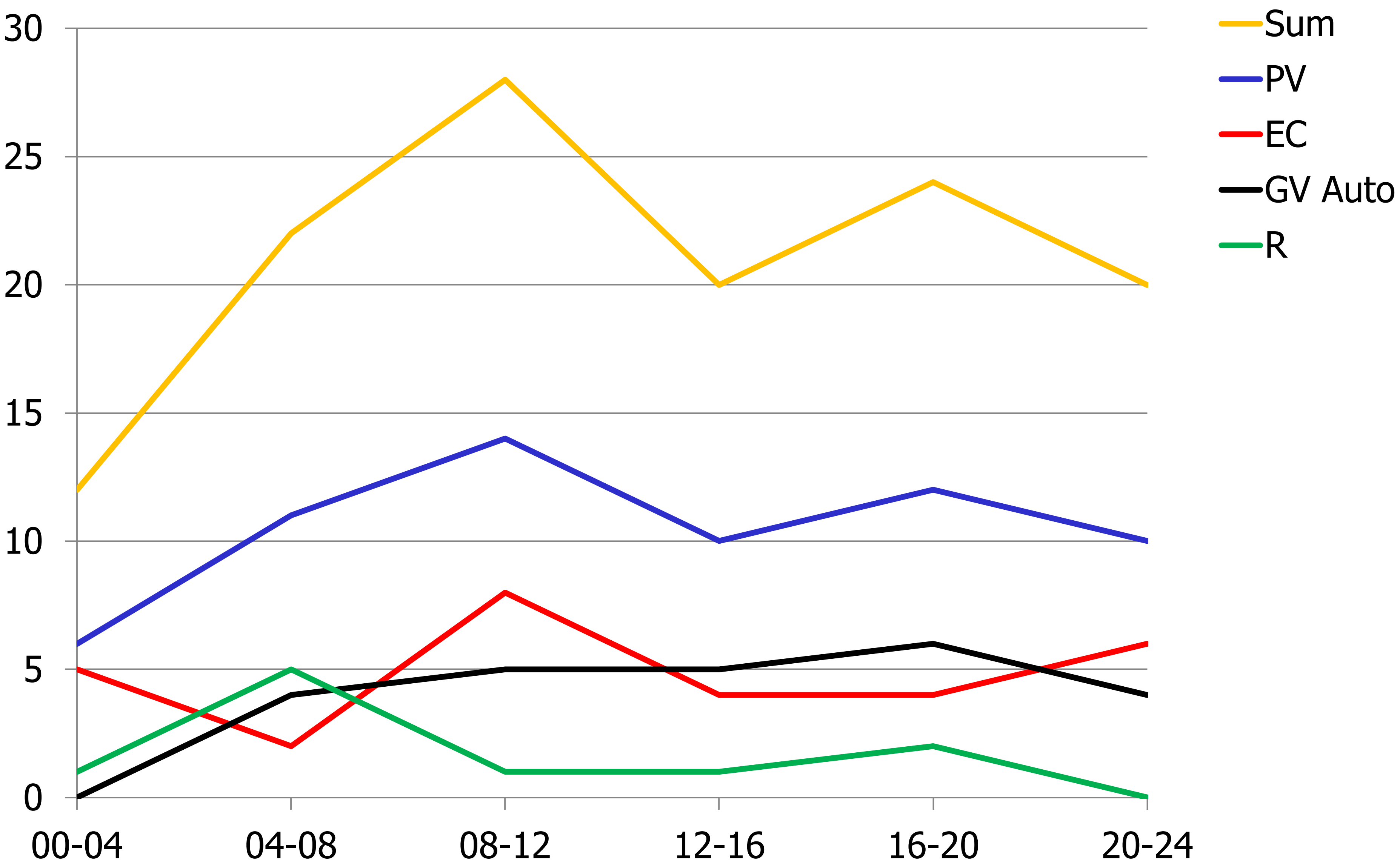
Crossing of trains

- complex crossings of AUTO trains in Iselle

Conflicting routes

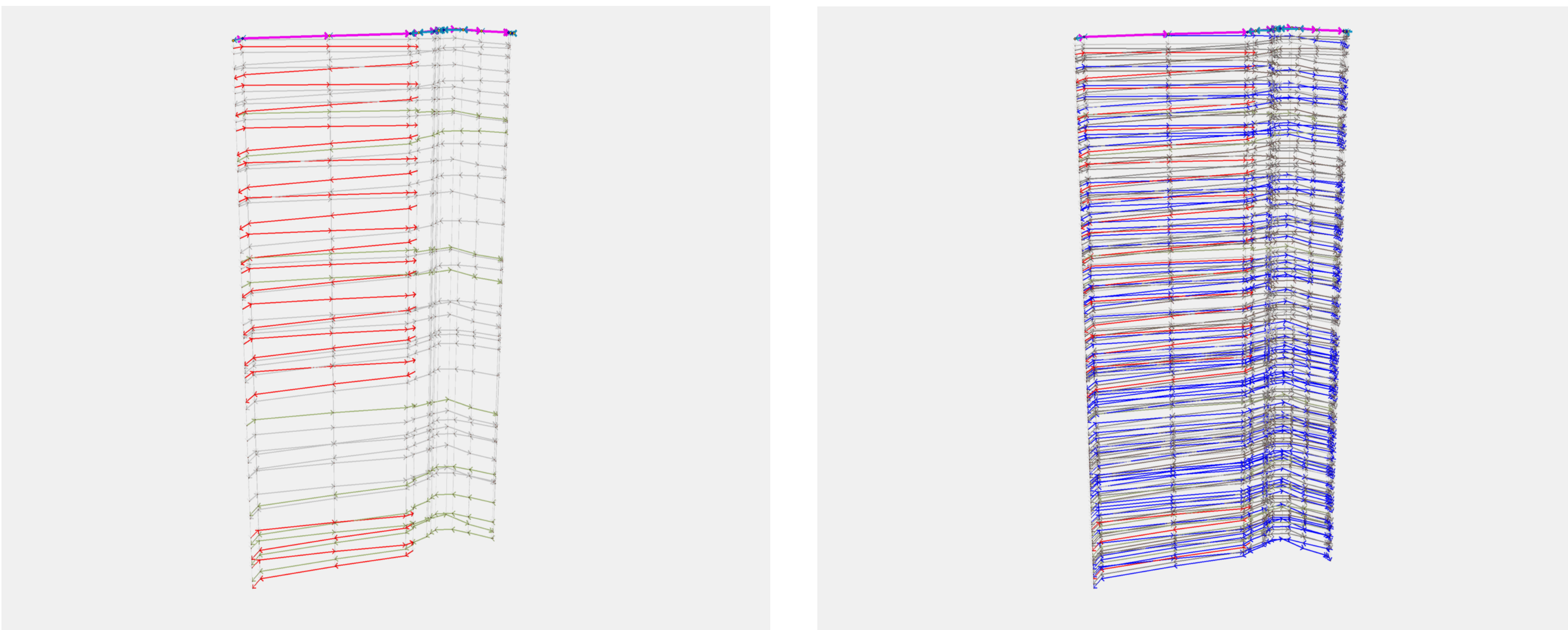
- complex routings in station area Domodossola and Brig

Dense Traffic at the Simplon



Estimation of the maximum theoretical corridor capacity

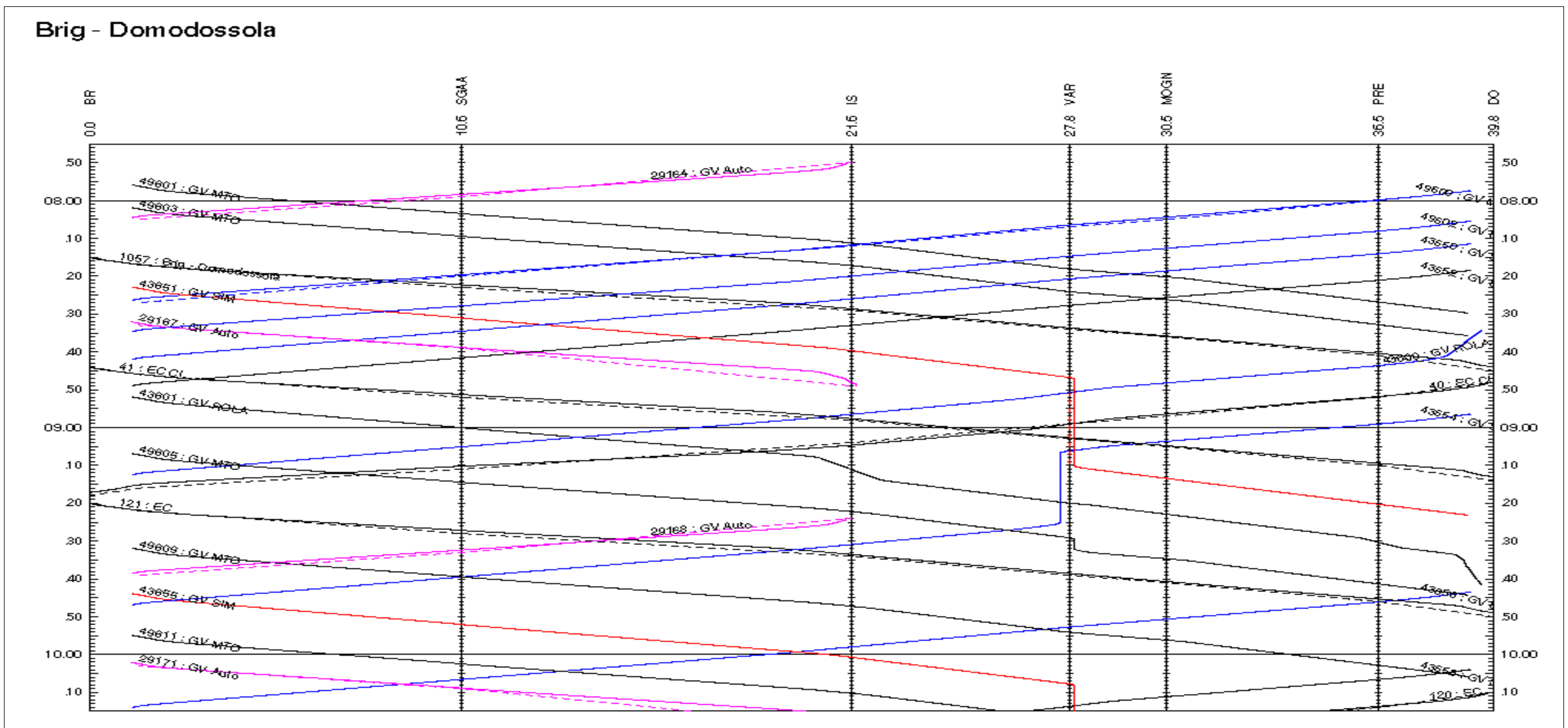
- Network accuracy of 6s
- Consider complete routing through stations
- Saturate by additional cargo trains

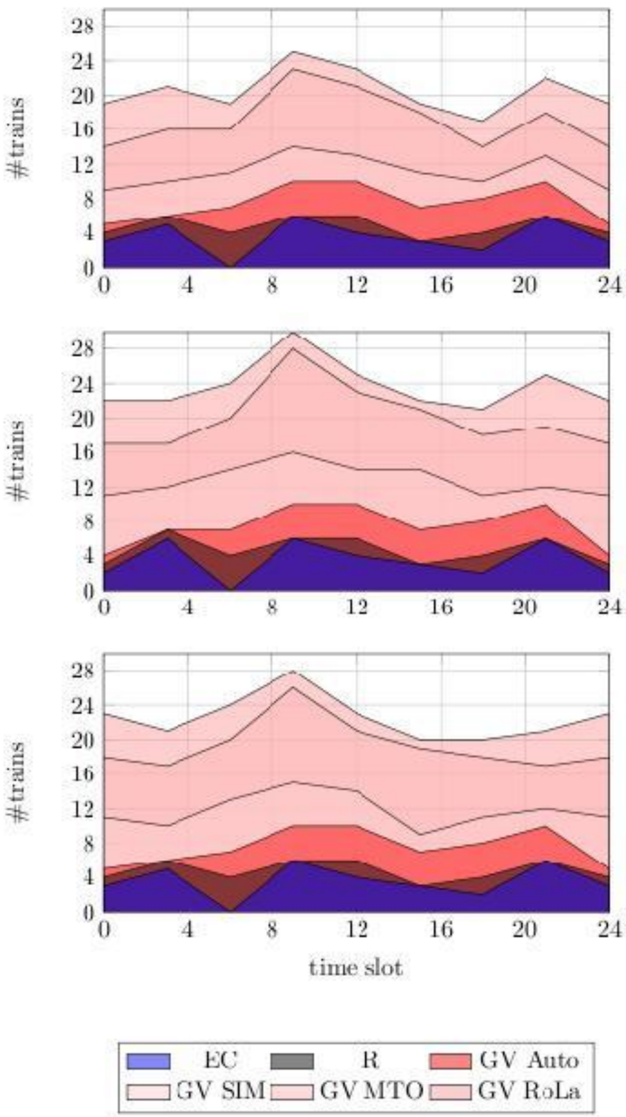


- Conflict free train schedules in simulation software (1s accuracy)

Aggregation-Test (Micro->Macro->Micro)

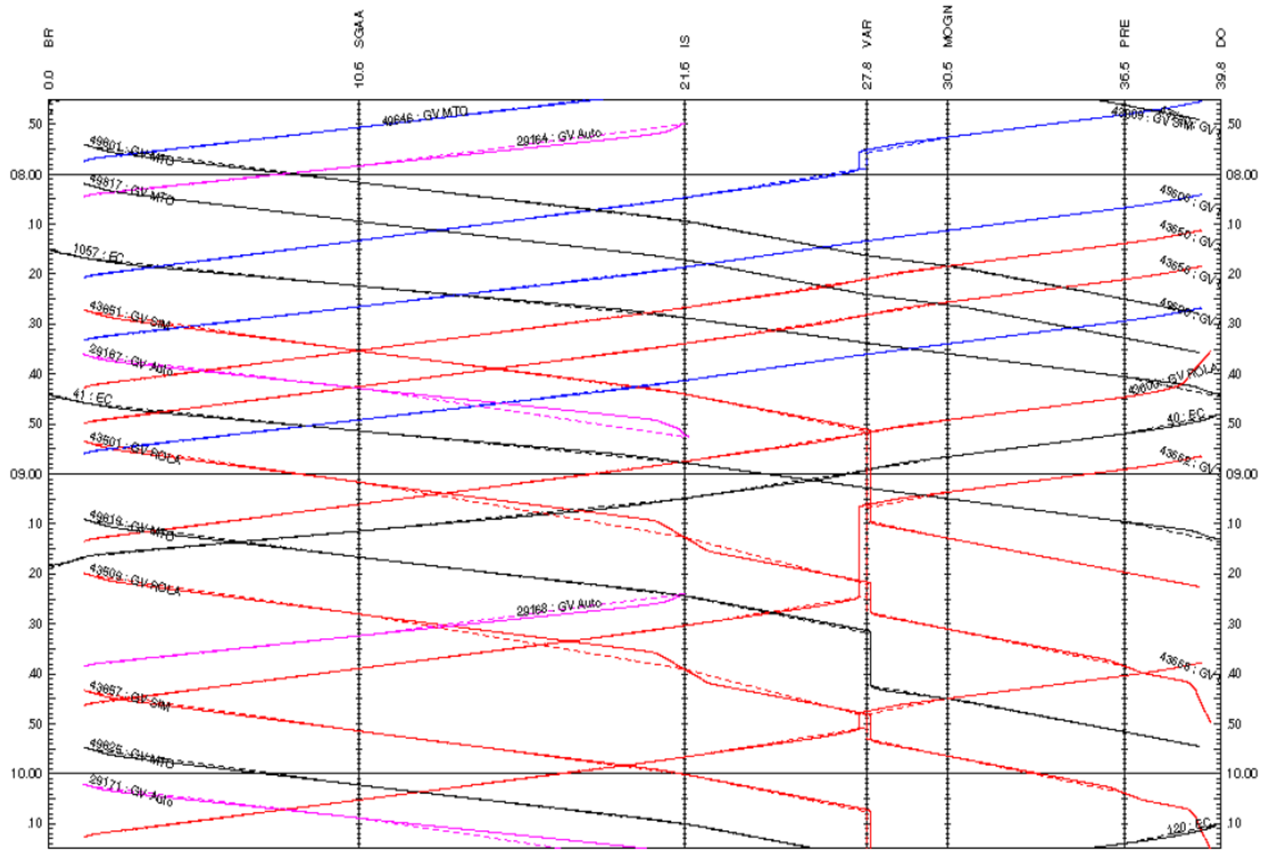
- Microscopic feasible 4h (8:00-12:00) reference plan in Open Track
- Reproducing this plan by an Optimization run
- Reimport to Open Track



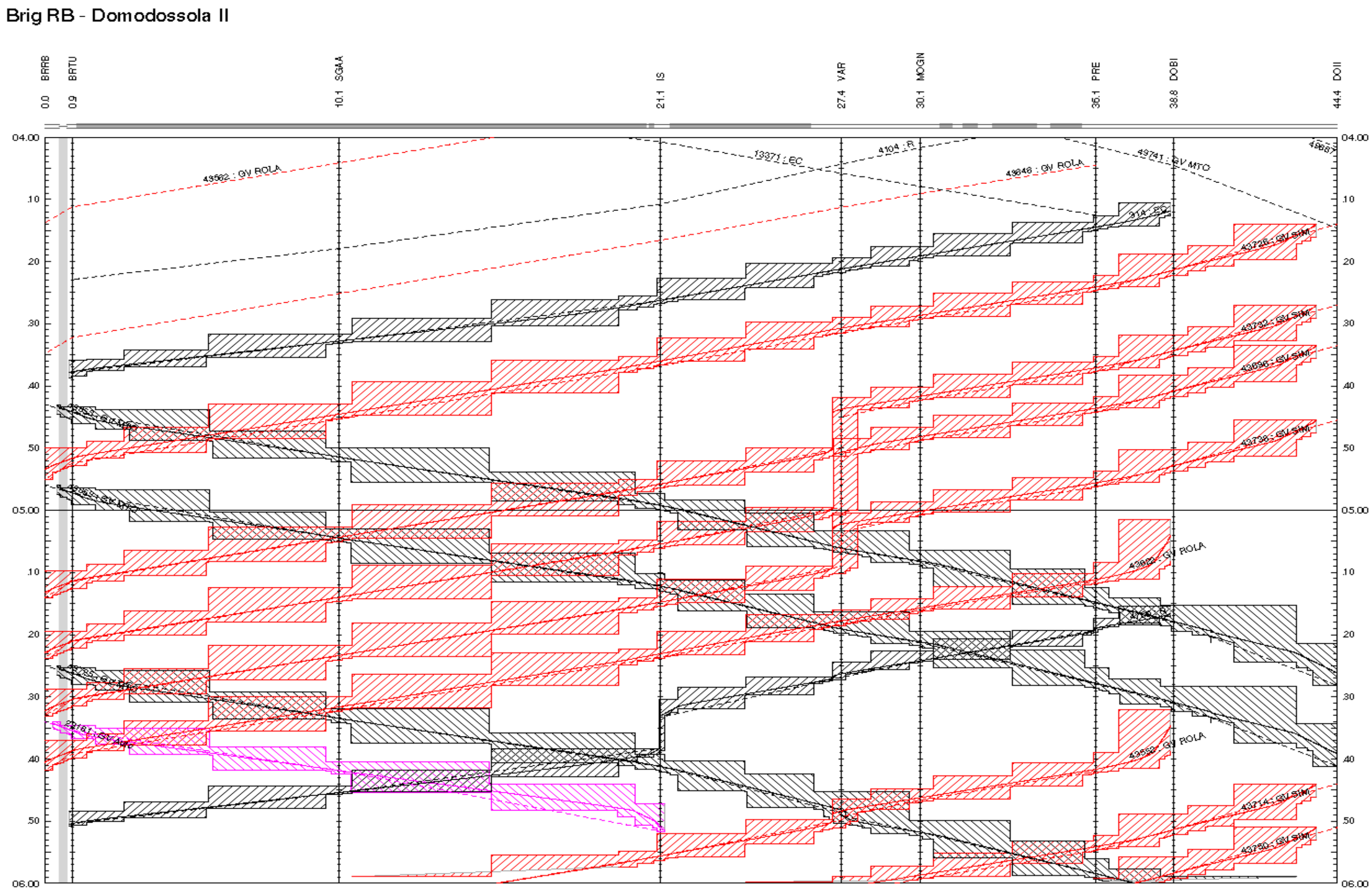


- 180 trains for network small (without station routing and buffer times)
- 196 trains for network big with precise routing through stations (without buffer times)
- 175 trains for network big with precise routing through stations and buffer times

- No delays, no early coming
- Feasible train routing and block occupation
- Timetable is valid in micro-simulation



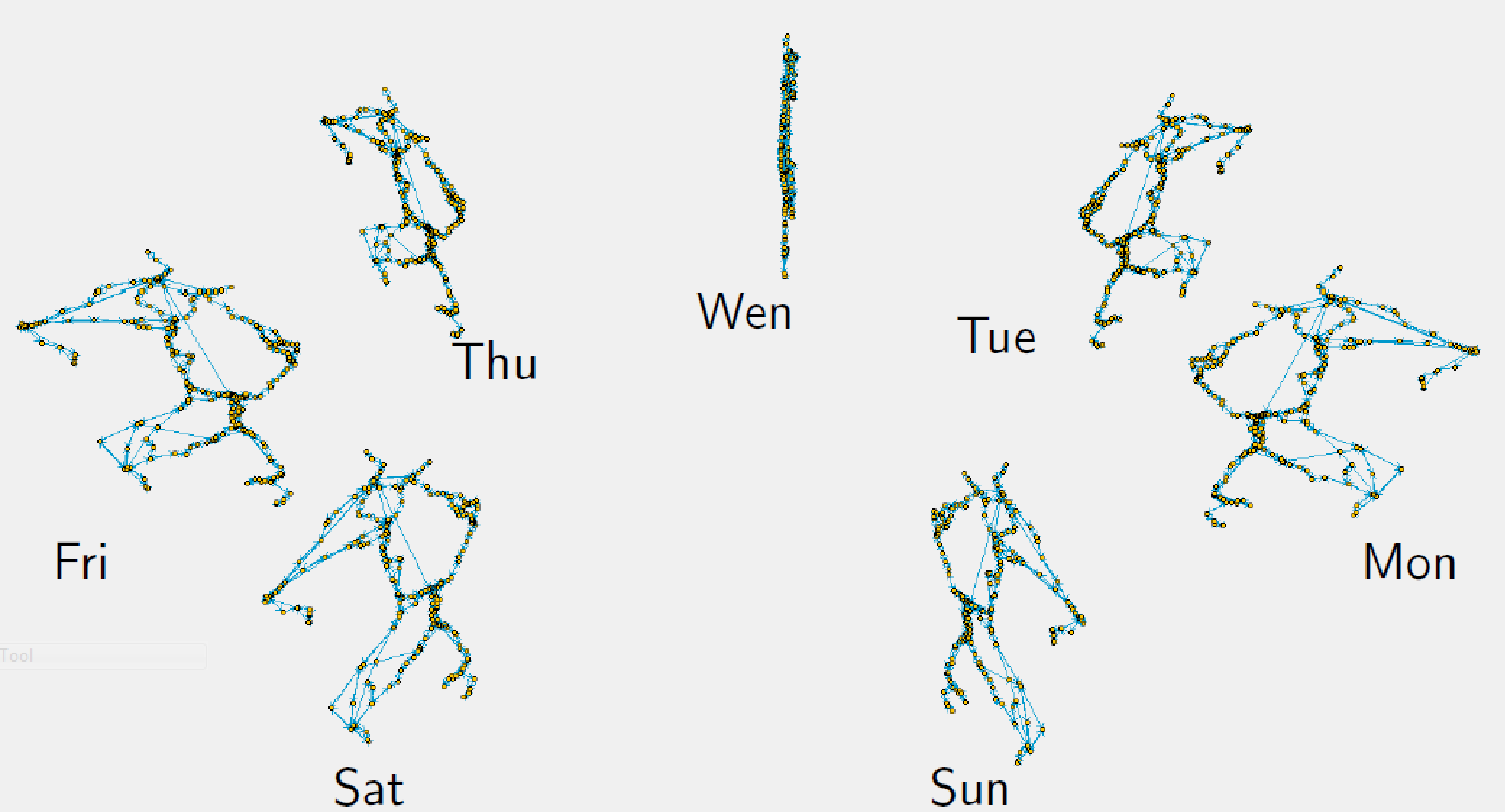
- Network big with buffer times



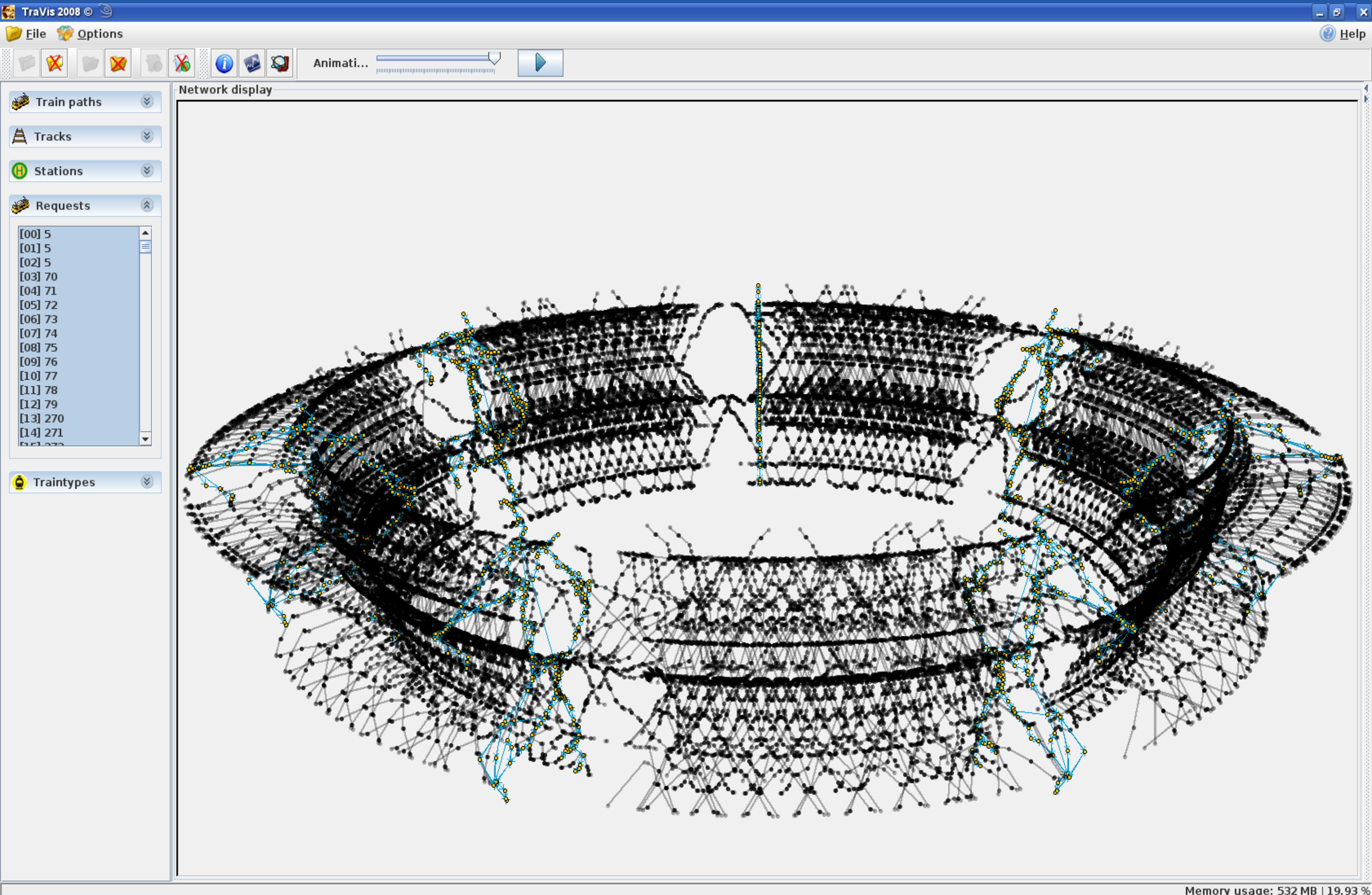
- Network big with buffer times

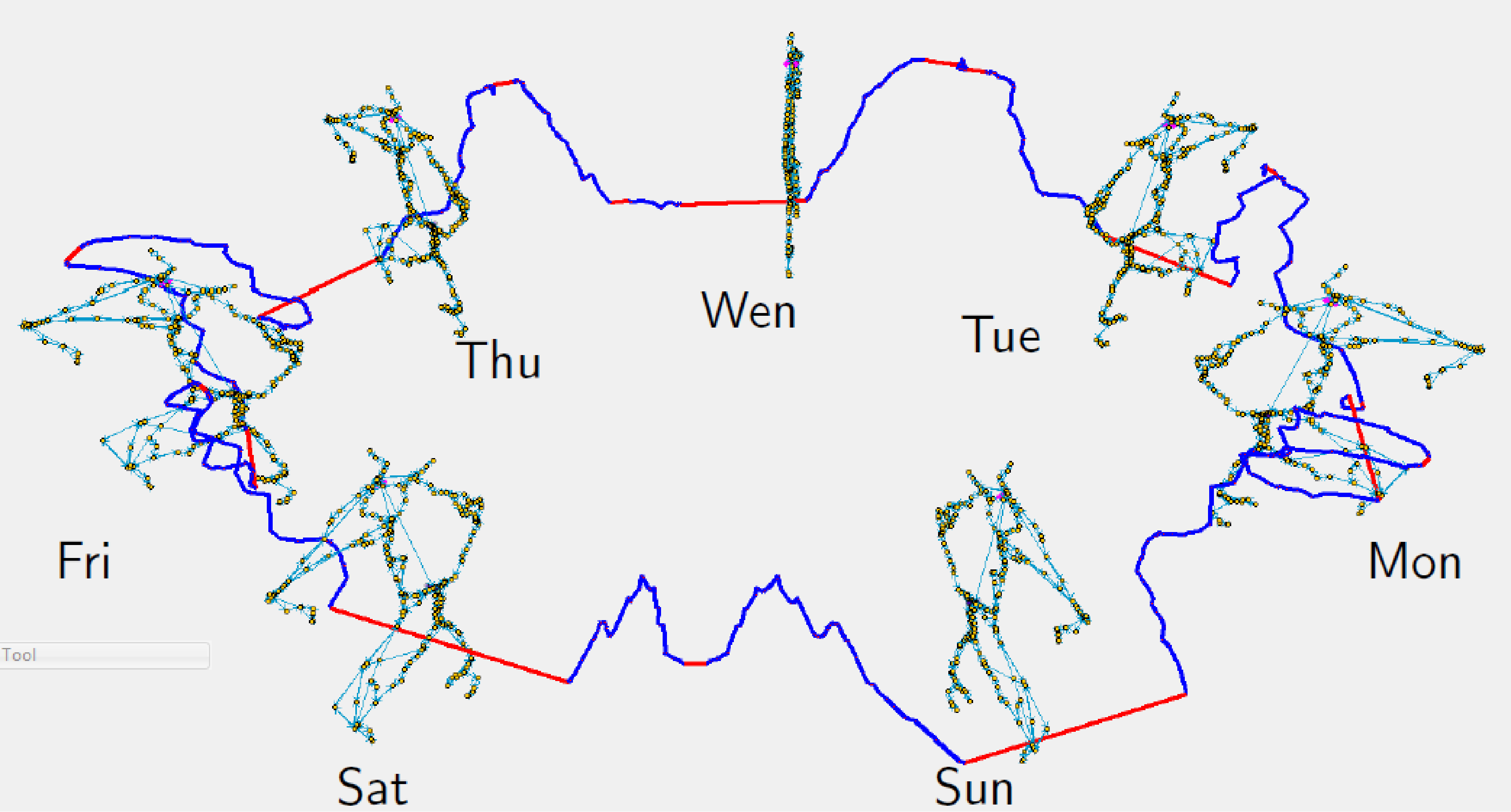
Time discretization dt/s	6	10	30	60
Number of trains	196	187	166	146
Cols in IP	504314	318303	114934	61966
Rows in IP	222096	142723	53311	29523
Solution time in secs	72774.55	12409.19	110.34	10.30

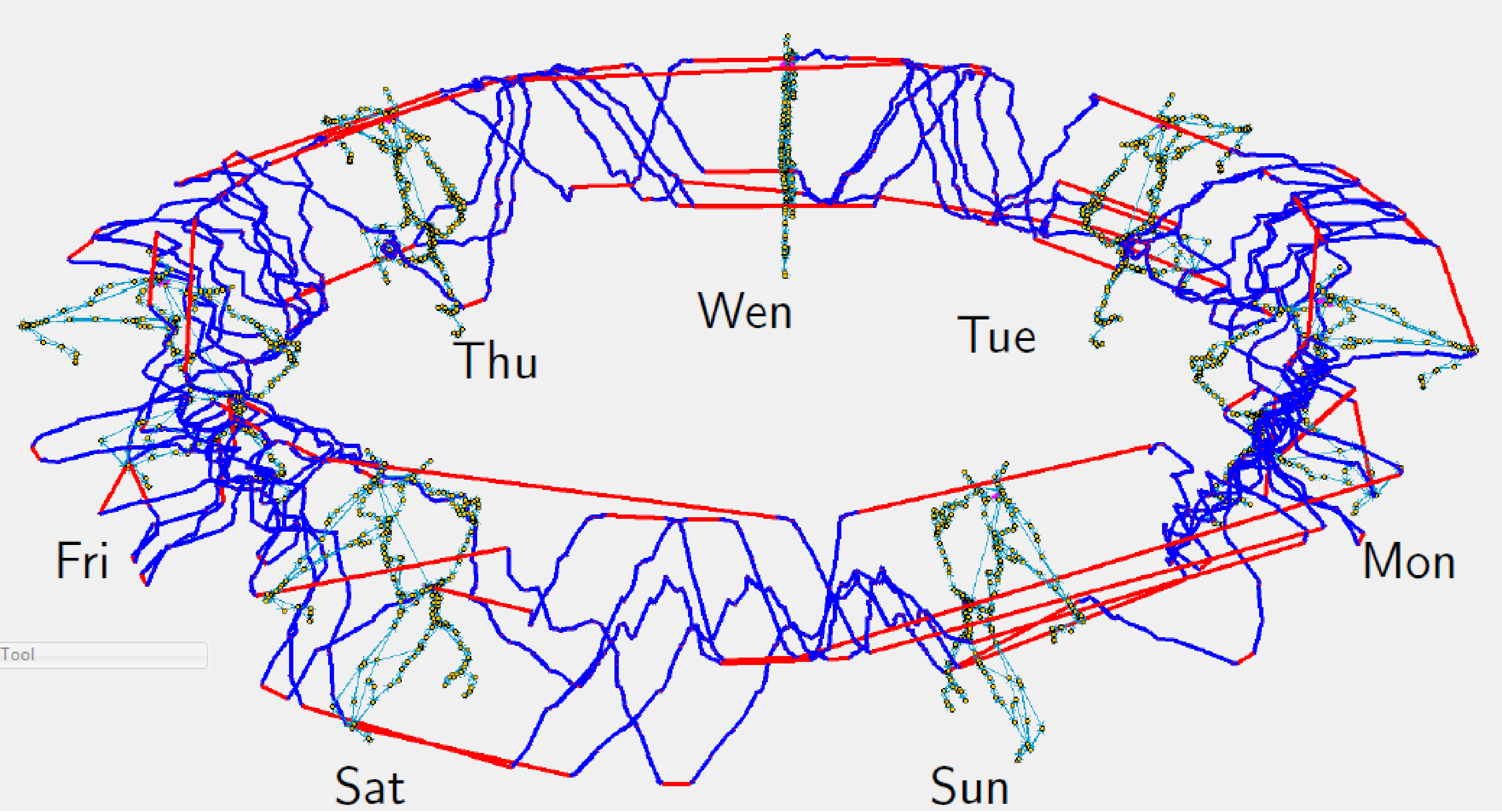
Hypergraph Scheduling


Wen
Thu
Tue
Fri
Mon
Sat
Sun
Tool

Cyclic Timetable for Standard Week

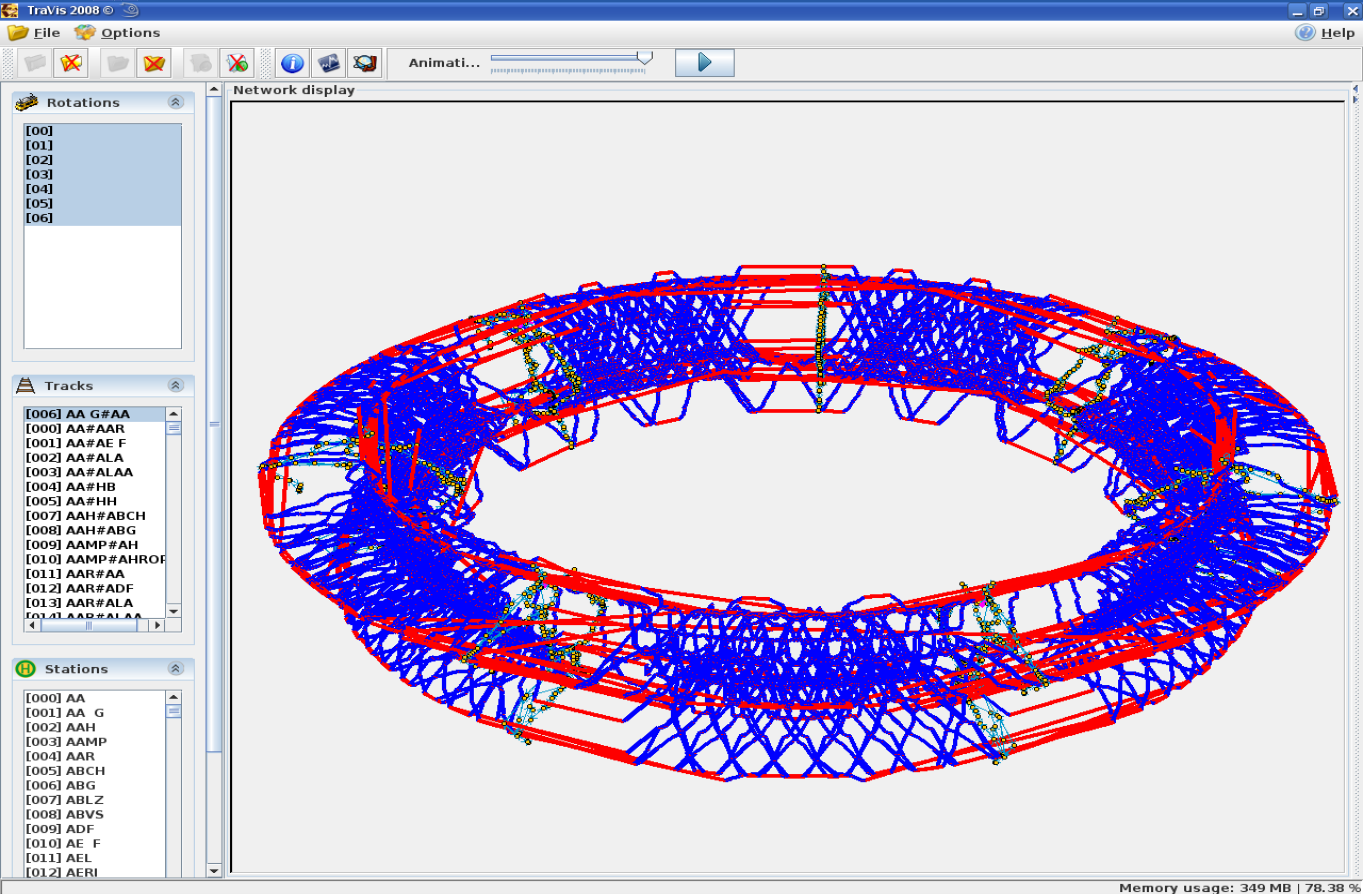



Wen
Thu
Tue
Fri
Mon
Sat
Sun
Tool


Wen
Thu
Tue
Fri
Mon
Sat
Sun
Tool

Rotation Schedule

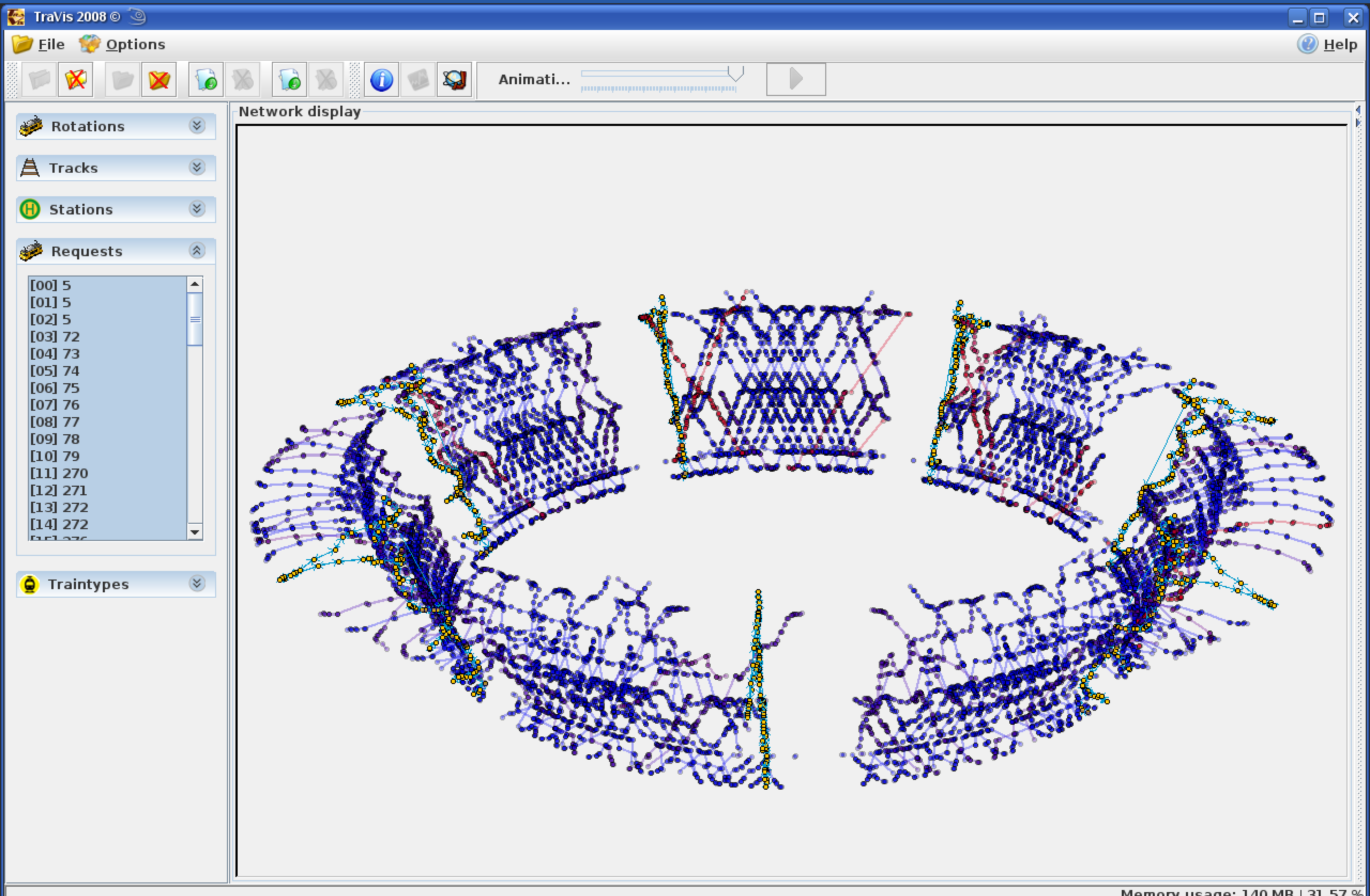
(Blue: Timetable, Red: Deadheads)




Wagenstandanzeiger Gleis 11
Zeit
Zug
Richtung
G
F
E
D
C
B
A
00.34 EN
05.36 IC
06.21 ICE
Köln / Bonn Flughafen
Köln
06.40 IC
07.45 IC
Bremen
07.45 IC
Bremen
08.45 IC
Oldenburg
09.40 IC
Düsseldorf
10.45 IC
Norddeich Mole
11.40 IC
Dortmund
12.45 IC
Oldenburg
13.40 IC
Düsseldorf
14.45 IC
Oldenburg
15.31 ICE
Köln / Bonn Flughafen
Köln
16.45 IC
Norddeich Mole
17.40 IC
Köln
18.45 IC
Oldenburg
19.40 IC
Dortmund
20.45 IC

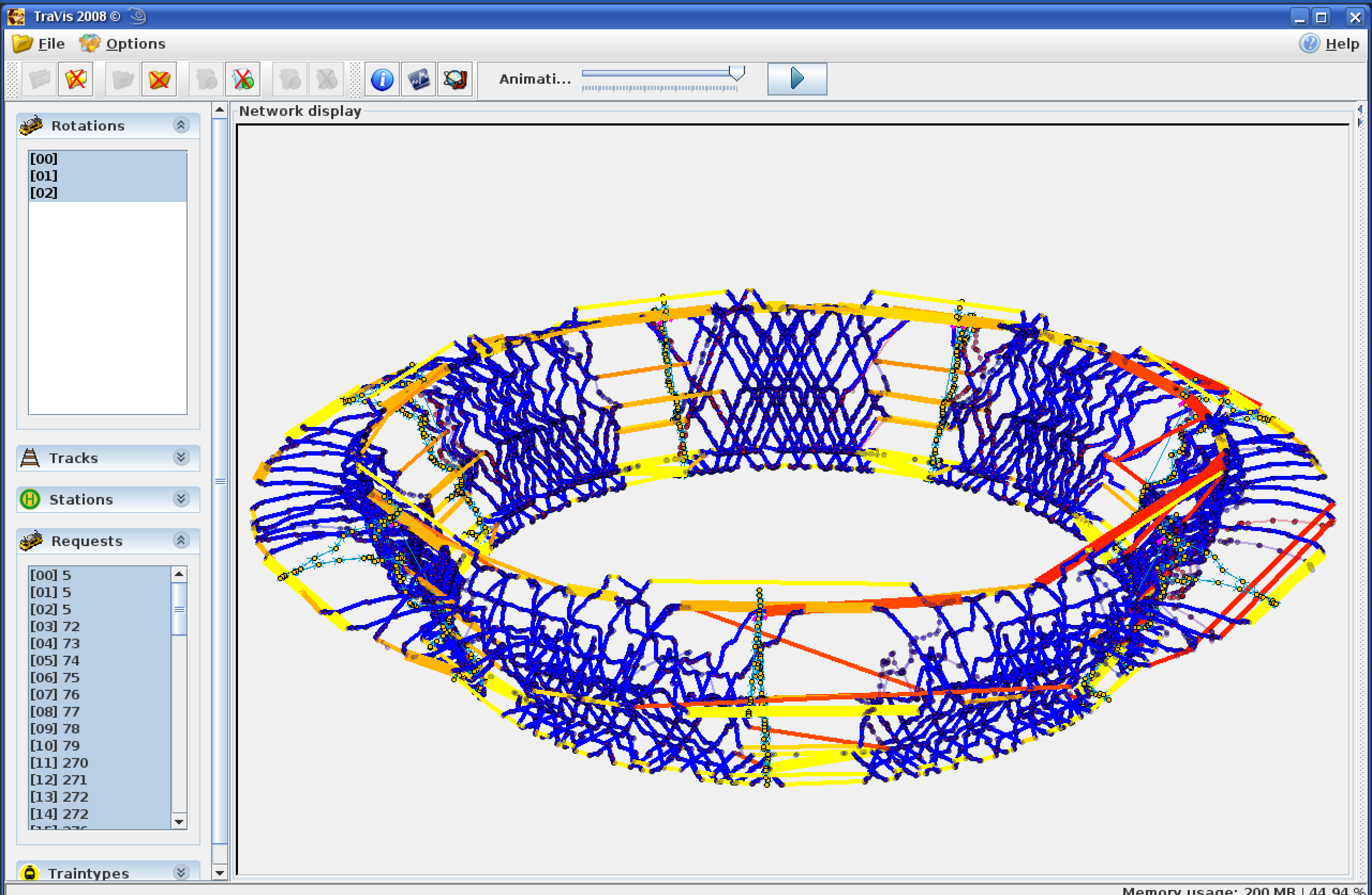
Uniformity

(Blue: Uniform, ..., Red: Irregular)

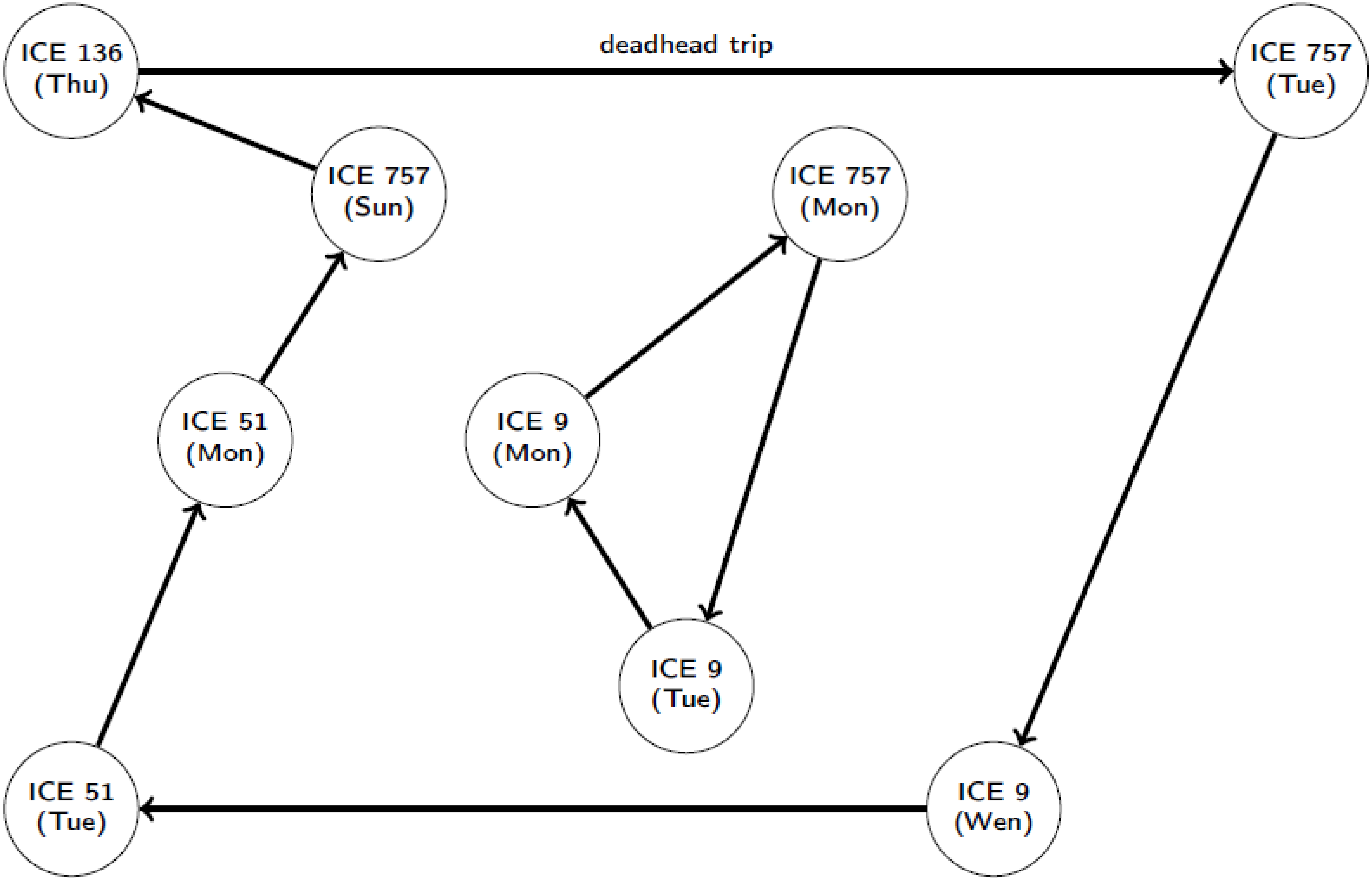


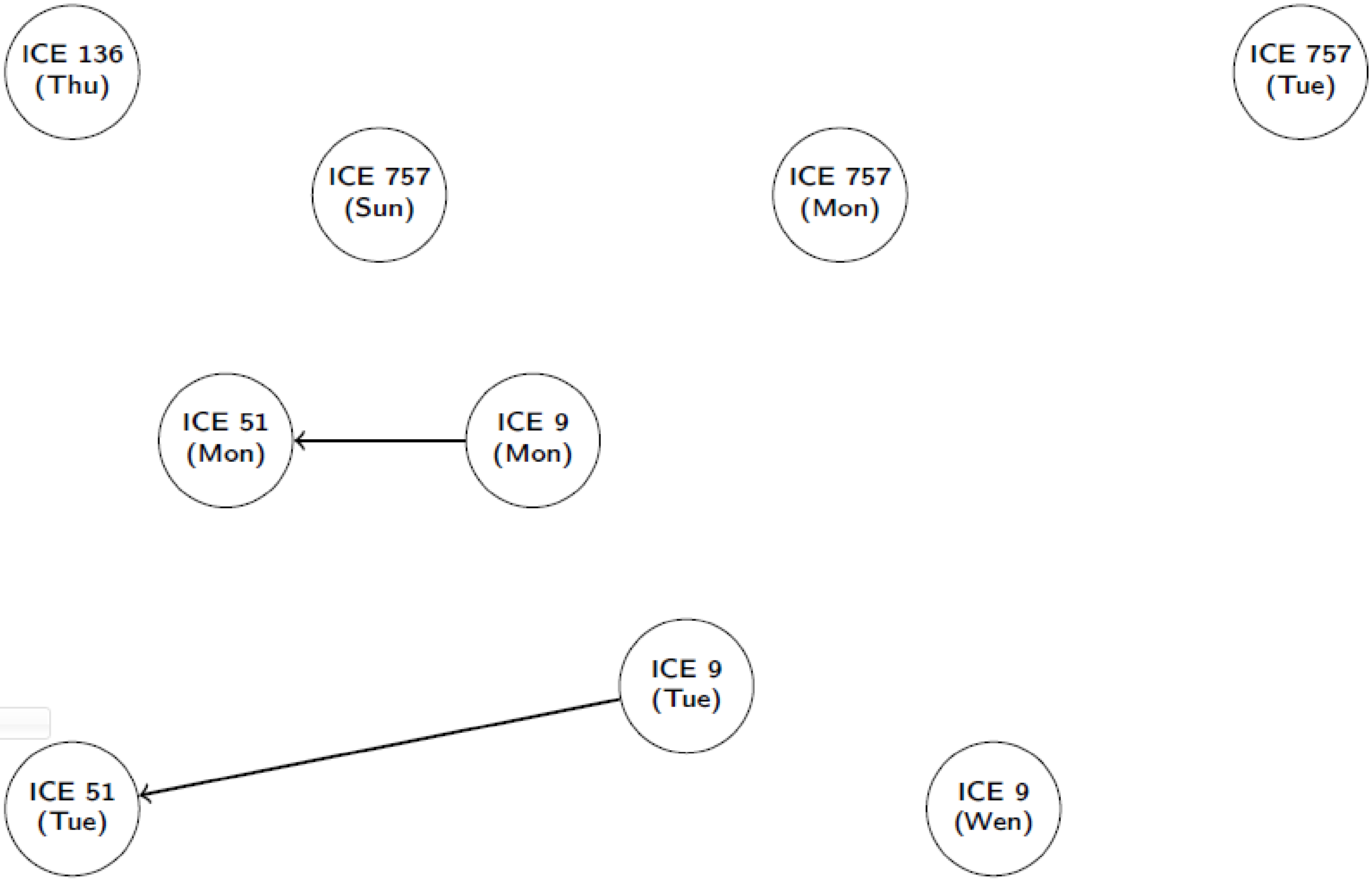
Uniformity

(Blue/Yellow: Uniform, ..., Red: Irregular, Fat: Maintenance)

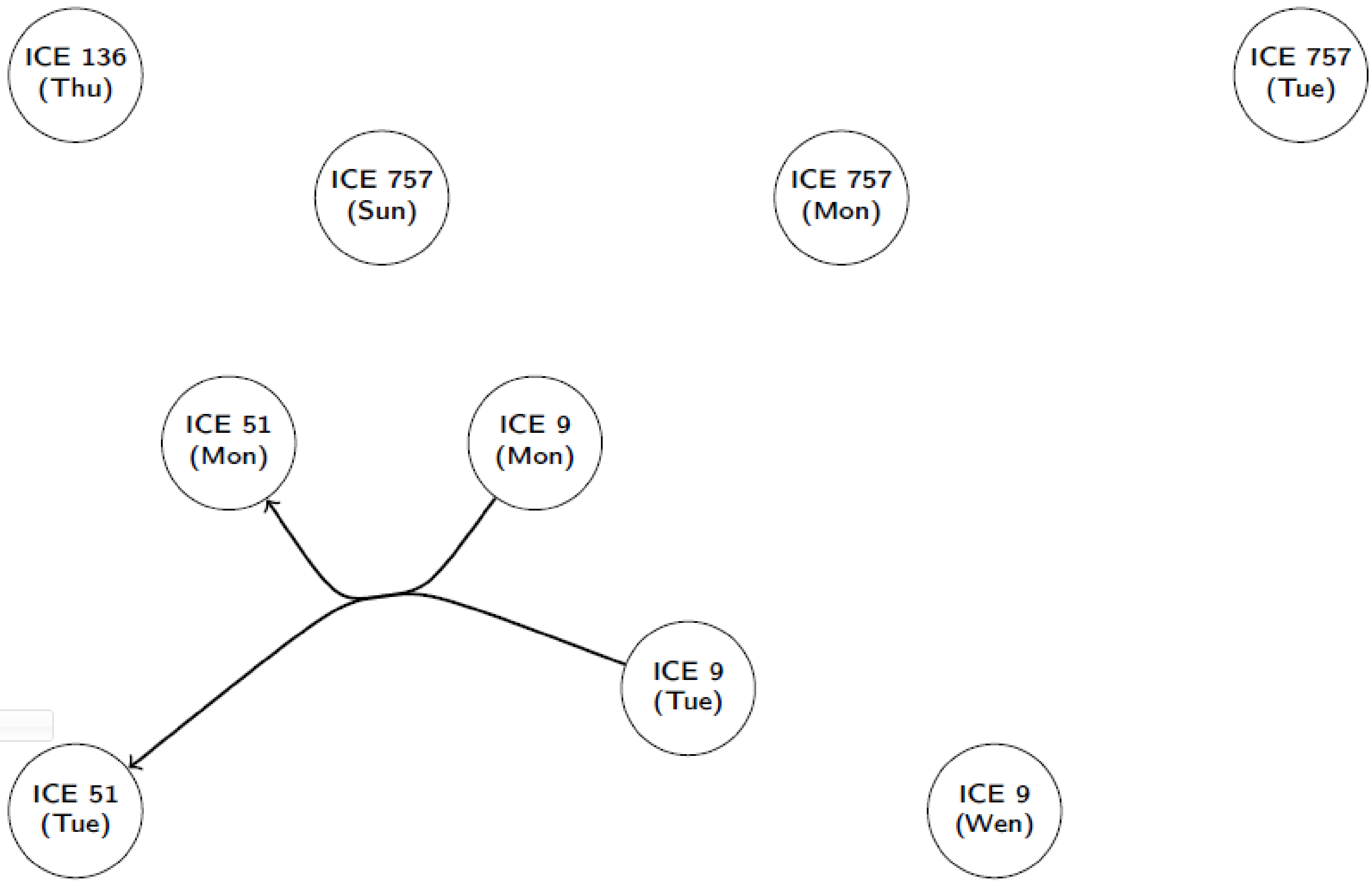


Rotation Schedule

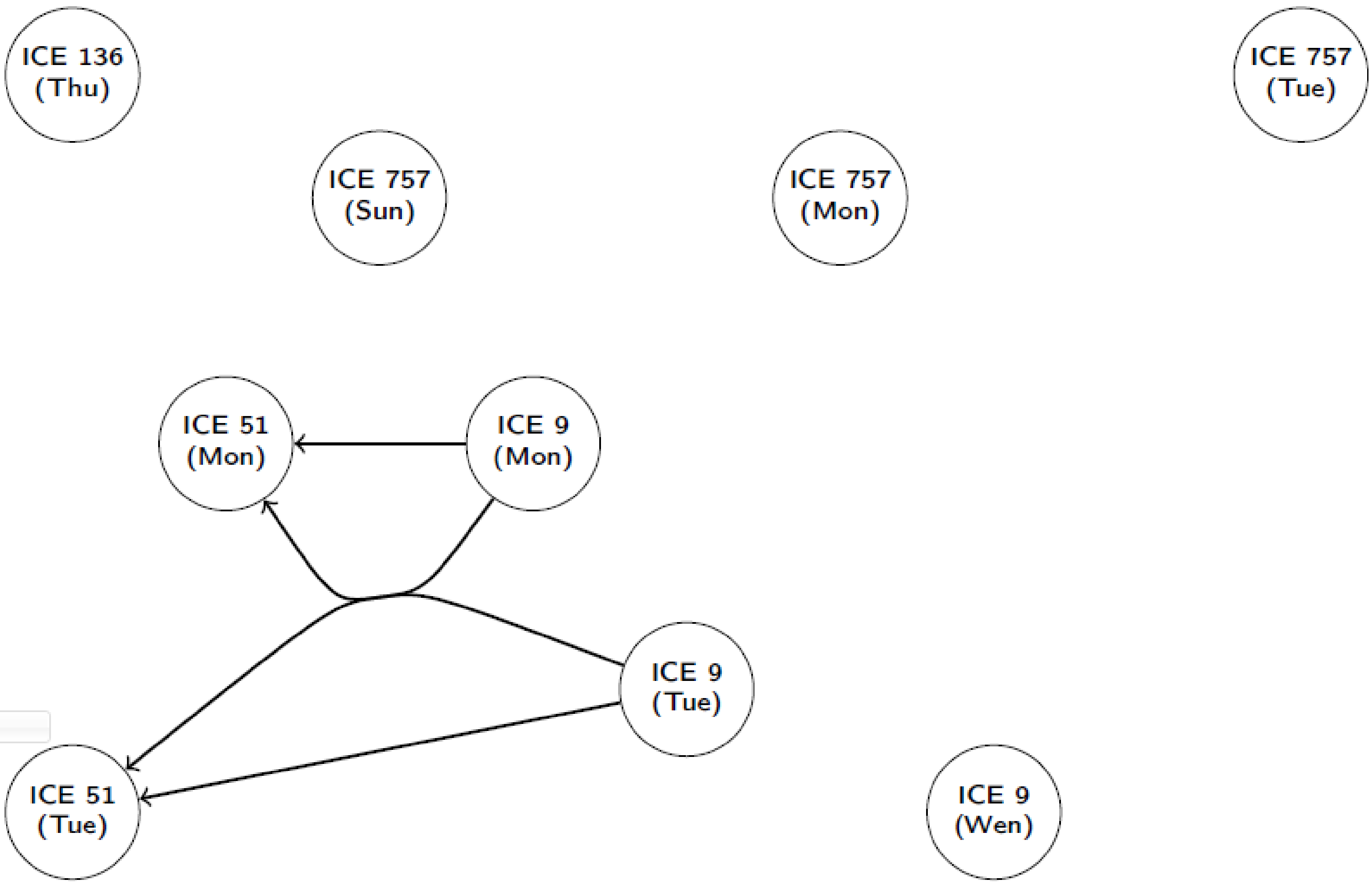


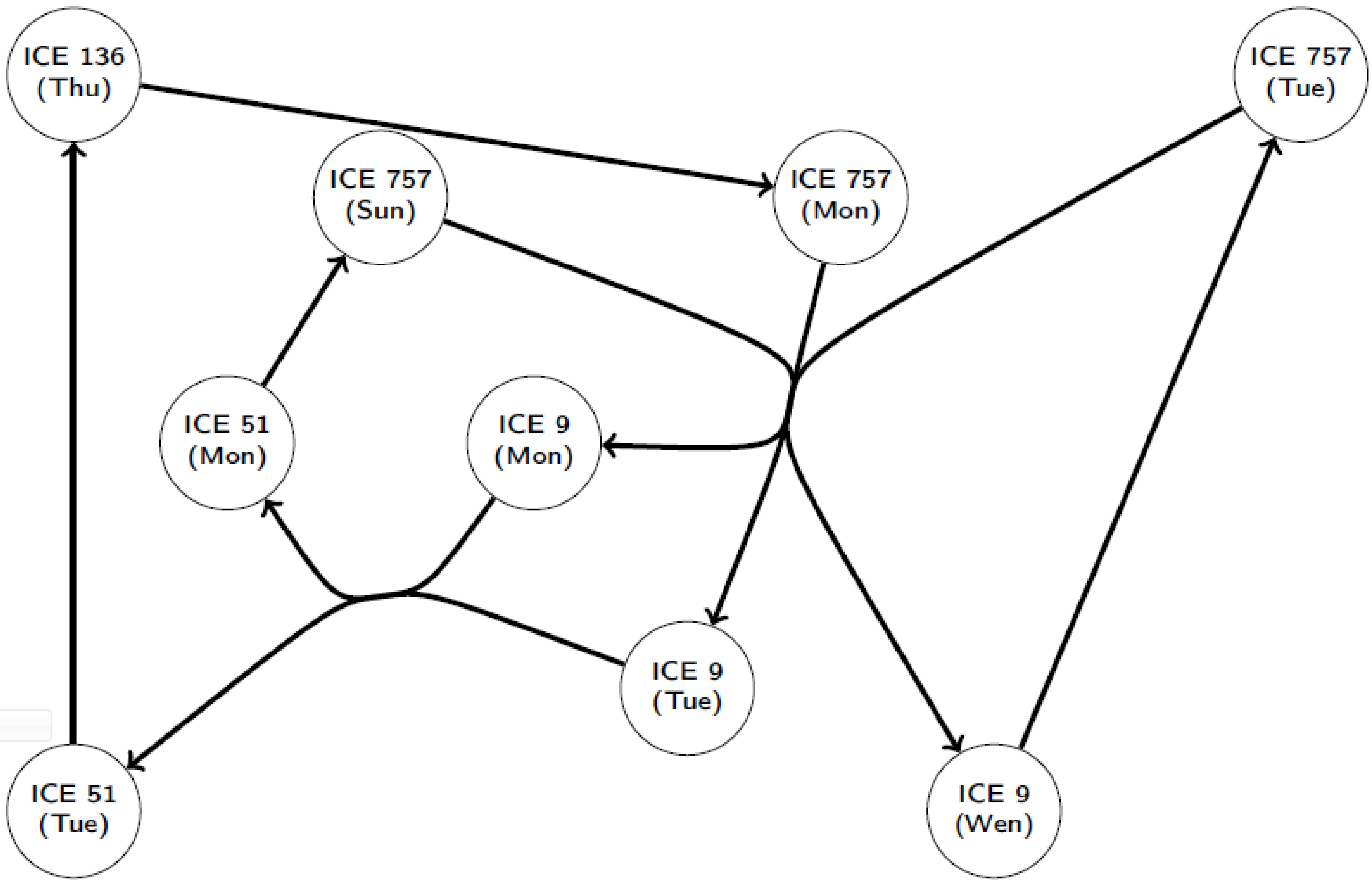

ICE 136
(Thu)
ICE 757
(Tue)
ICE 757
(Sun)
ICE 757
(Mon)
ICE 51
(Mon)
ICE 9
(Mon)
ICE 9
(Tue)
ICE 51
(Tue)
ICE 9
(Wen)

Uniformity


ICE 136
(Thu)
ICE 757
(Tue)
ICE 757
(Sun)
ICE 757
(Mon)
ICE 51
(Mon)
ICE 9
(Mon)
ICE 9
(Tue)
ICE 51
(Tue)
ICE 9
(Wen)

Modelling Uniformity Using Hyperarcs




ICE 136
(Thu)
ICE 757
(Tue)
ICE 757
(Sun)
ICE 757
(Mon)
ICE 51
(Mon)
ICE 9
(Mon)
ICE 9
(Tue)
ICE 51
(Tue)
ICE 9
(Wen)

Definition: Let D=(V,A) be a directed hypergraph w. arc costs c_a

- ▷ H⊆A hyperassigment :⇔ $\delta^+(v) \cap H = \delta^-(v) \cap H = 1$
- ▷ Hyperassignment Problem :⇔ argmin c(H), H hyperassignment

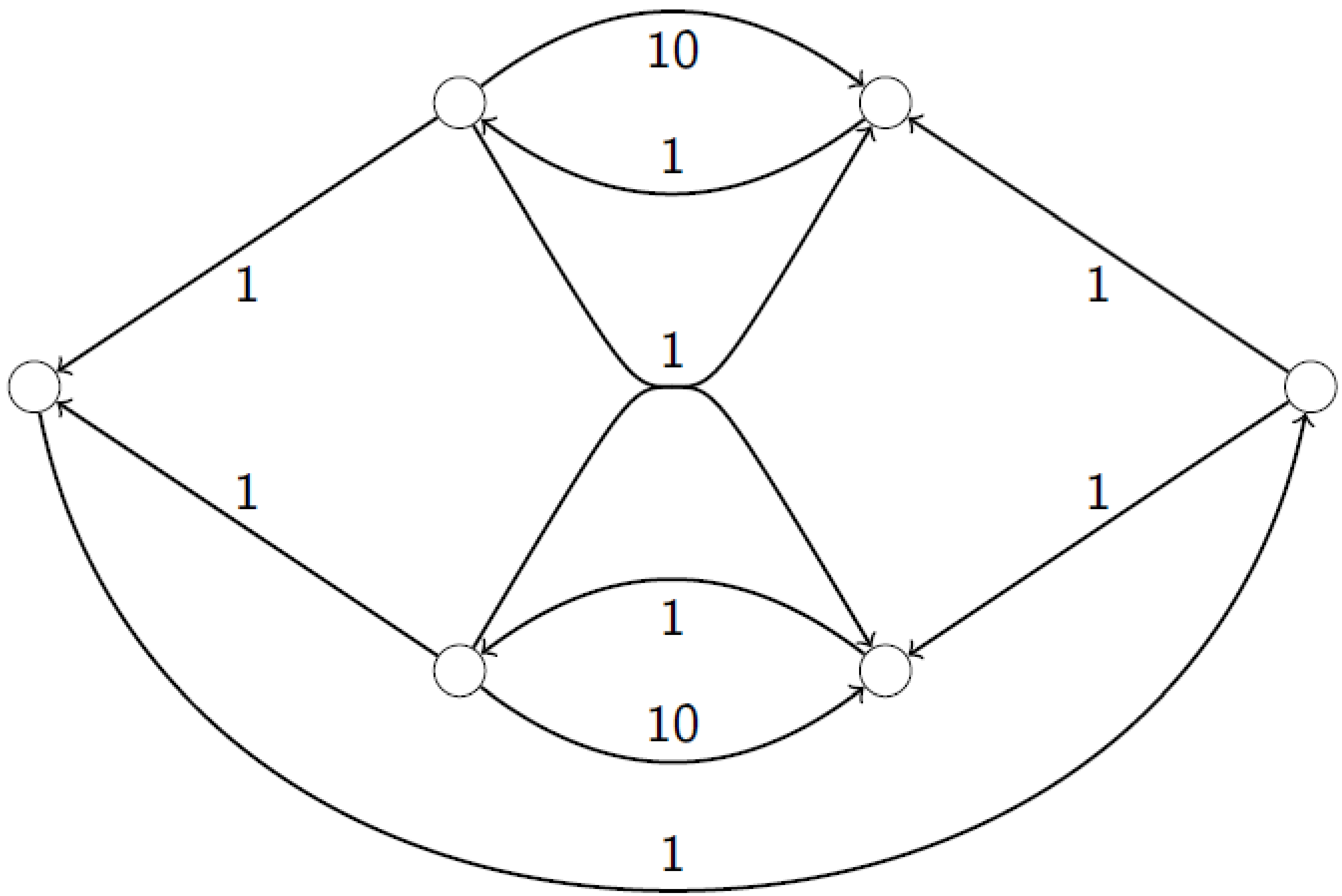
$$
\begin{array}{llcll}
\min & c^T x & & & \\
& x(\delta^+(v)) & = & 1 & \forall v \in V \\
& x(\delta^-(v)) & = & 1 & \forall v \in V \\
& x & \in & \{0,1\}^A &
\end{array}
$$

Literature

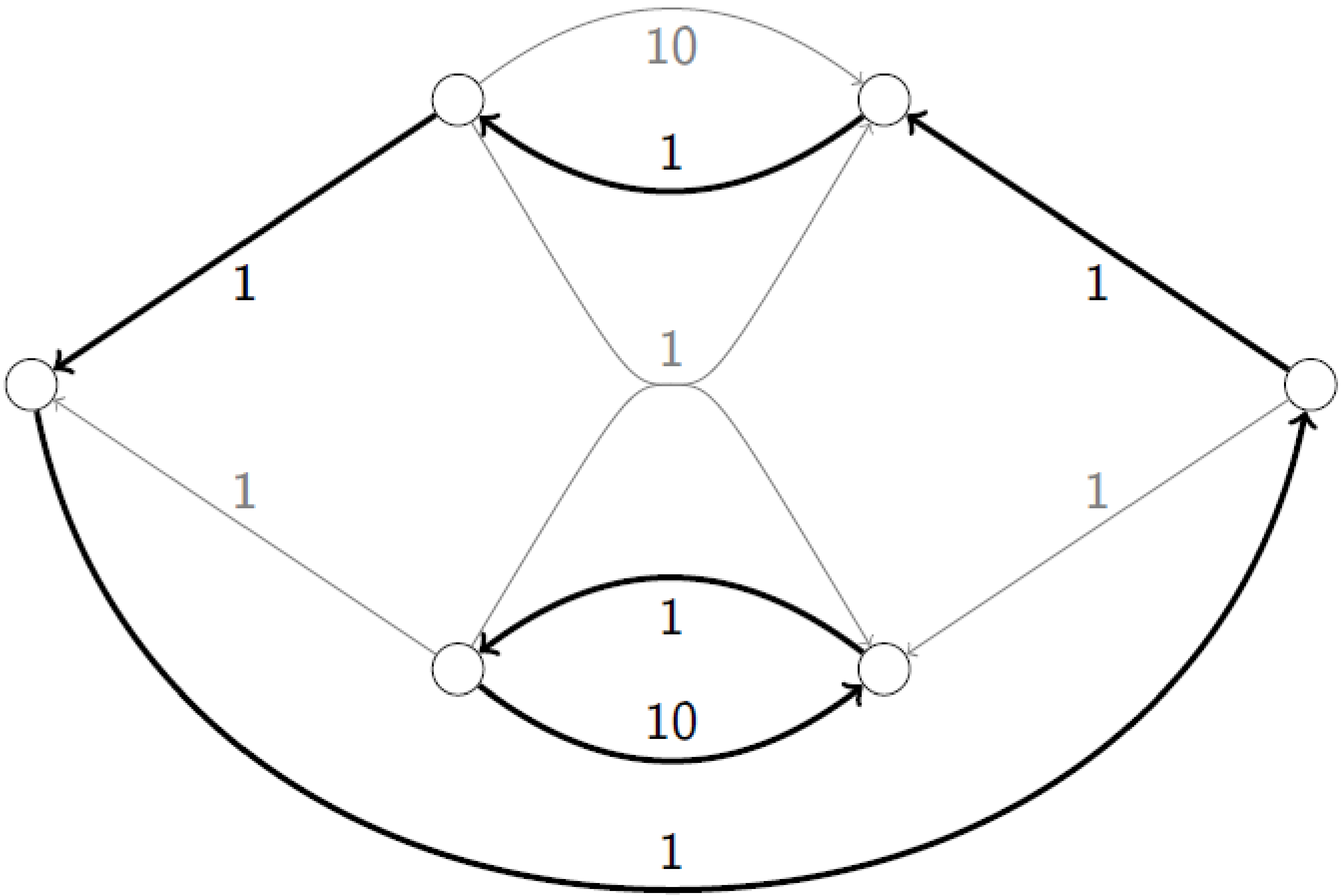
- ▷ Cambini, Gallo, Scutellà (1992): Minimum cost flows on hypergraphs; solves only the LP relaxation
- ▷ Jeroslow, Martin, Rarding, Wang (1992): Gainfree Leontief substitution flow problems; does not hold for the hyperassignment problem

Theorem: The HAP is NP-hard (even for simple cases).

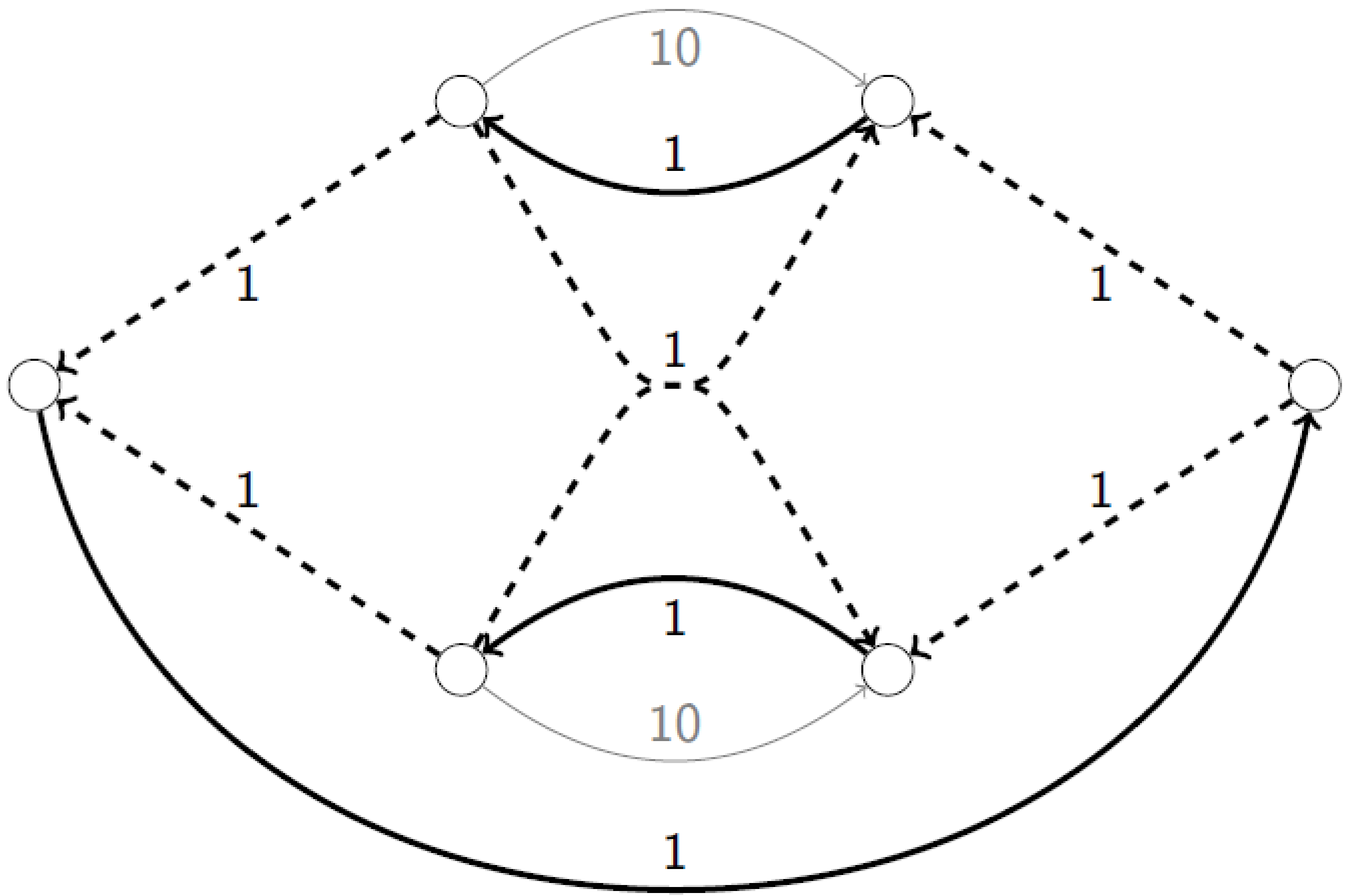
Theorem: The LP/IP gap of HAP can be arbitrarity large.



Theorem: The LP/IP gap of HAP can be arbitrarity large.



Theorem: The LP/IP gap of HAP can be arbitrarity large.



Theorem: The LP/IP gap of HAP can be arbitrarity large.

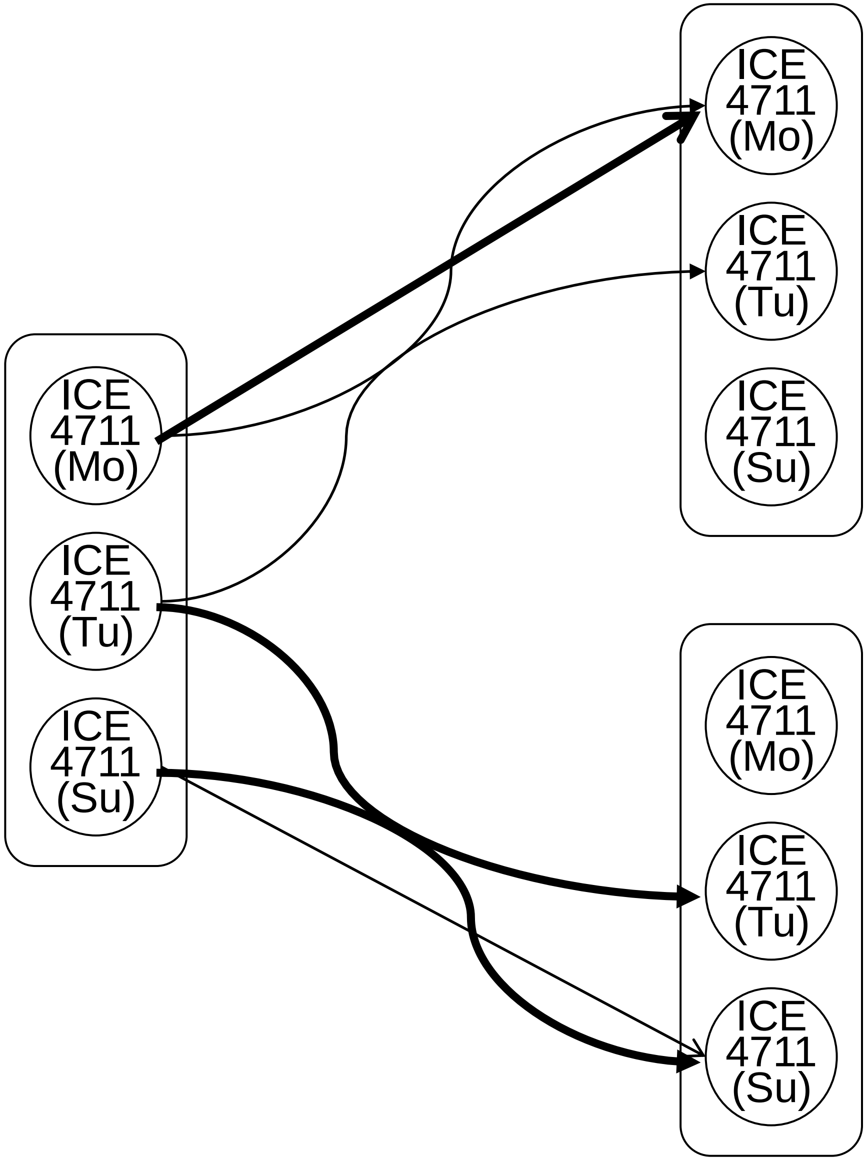
Proposition: The determinants of basis matrices of HAP can be arbitrarily large, even if all hyperarcs have head and tail size 2.

Proposition: HAP is APX-complete for hyperarc head and tail size 2 in general and for hyperarc head and tail cardinality 3 in the revelant cases.

# rows (2 · |V|)	# columns (|A|)	nonzeros	LP-IP gap	root gap	root improvement	# clique cuts	# other cuts	root run time (sec.)
534	52056	140081	11.16 %	6.81 %	4.90 %	160	14	8
620	80477	236020	8.72 %	0.00 %	9.54 %	120	2	29
812	102375	216566	0.38 %	0.18 %	0.20 %	24	16	40
1128	267542	732134	4.59 %	0.26 %	4.55 %	263	0	160
1310	363513	1006024	7.85 %	0.22 %	8.28 %	378	2	270
1496	469932	1369224	18.70 %	1.86 %	20.71 %	809	0	971
1696	618348	1787078	5.17 %	0.16 %	5.28 %	925	0	1705
1746	649525	1859898	7.52 %	4.88 %	2.86 %	563	0	1129
1798	647650	1822718	13.60 %	0.95 %	14.65 %	537	0	1099
1798	647650	1822718	13.35 %	0.62 %	14.69 %	604	0	873
2006	855153	2491372	5.76 %	0.68 %	5.39 %	1025	0	2490
2260	1079535	3138752	9.89 %	2.03 %	8.73 %	954	0	5483
2502	1290750	3680124	7.06 %	0.76 %	6.79 %	801	0	4583
2620	1432355	4187296	9.05 %	1.15 %	8.68 %	1068	0	7910
2624	1439453	4087042	14.17 %	5.23 %	10.41 %	951	0	(*) 14400

(*) = aborted

Partitioned Hypergraph and Configurations



Theorem: There is an extended formulation of HAP with O(V^8) variables that implies all clique constraints.

$$
\begin{array}{llcll}
\min & c^T x & & & \\
 & x(\delta^+(v)) & = & 1 & \forall v \in V \\
 & x(\delta^-(v)) & = & 1 & \forall v \in V \\
 & x & \in & \{0,1\}^A & \\
 & y(C^+(a)) & = & x_a & \forall a \in A \\
 & y(C^-(a)) & = & x_a & \forall a \in A \\
 & y & \in & \{0,1\}^C &
\end{array}
$$

Stochastic Scheduling

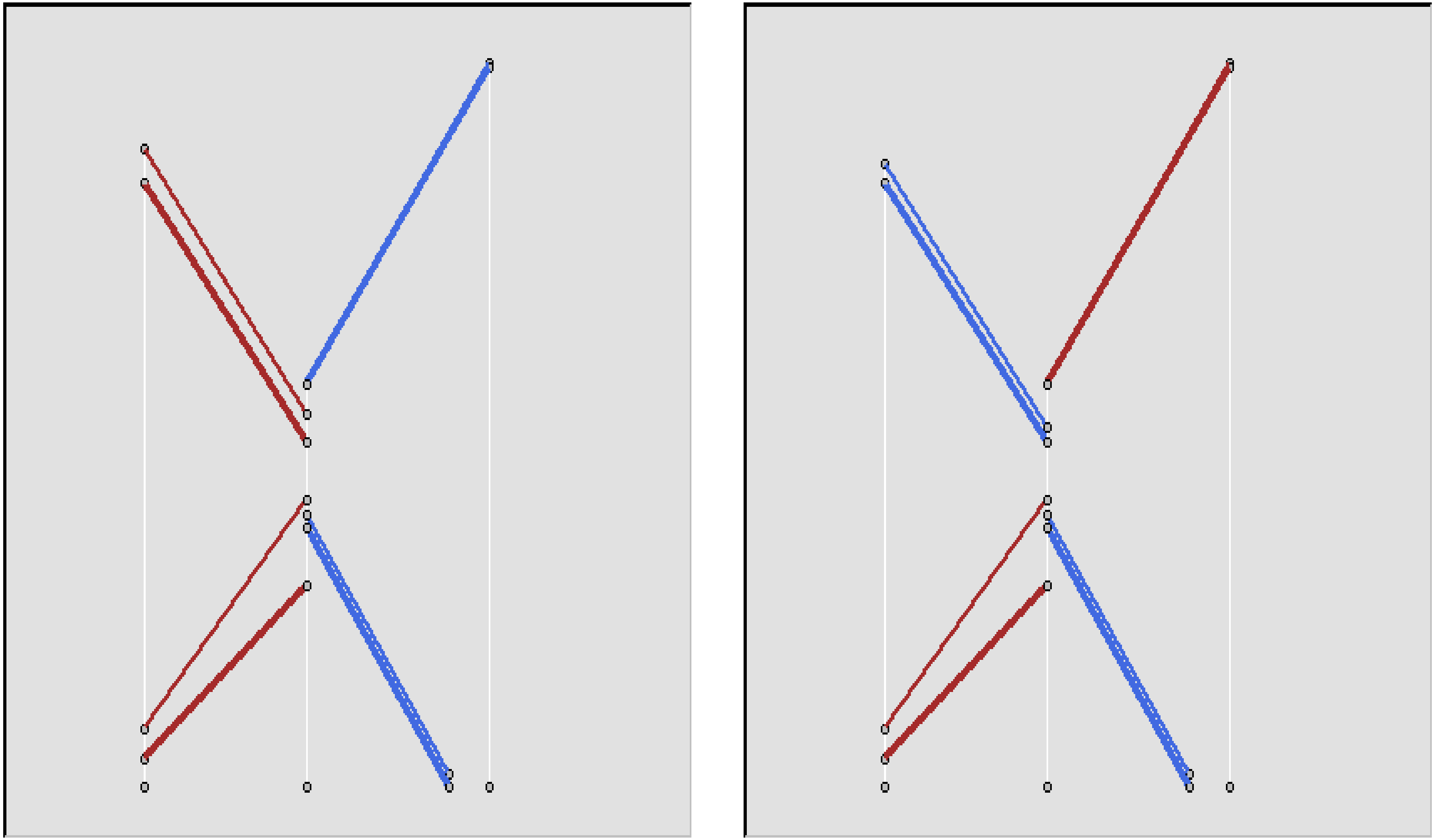
Cost of delays

- 72 €/minute average cost of gate delay over 15 minutes, cf. EUROCONTROL [2004]
- 840 – 1200 millions € annual costs caused by gate delays in Europe

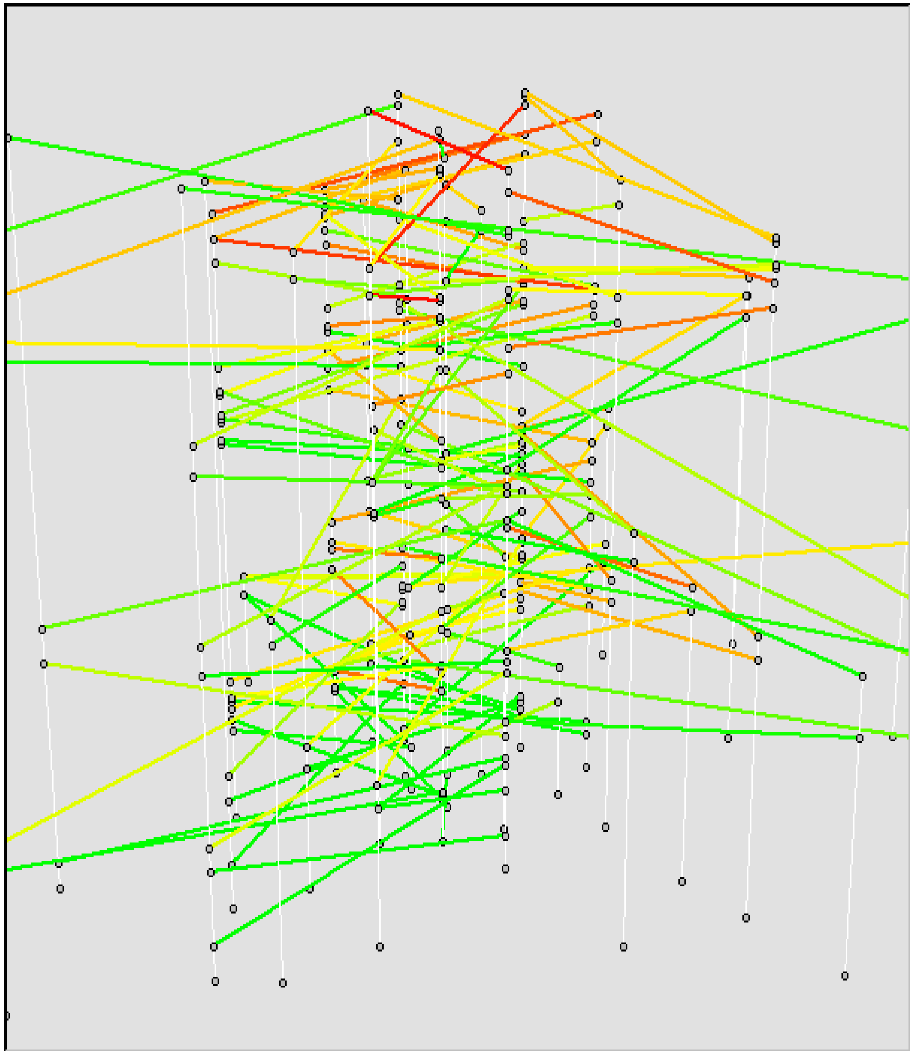
Benefits of robust planning

- Cost savings
- Reputation
- Less operational changes

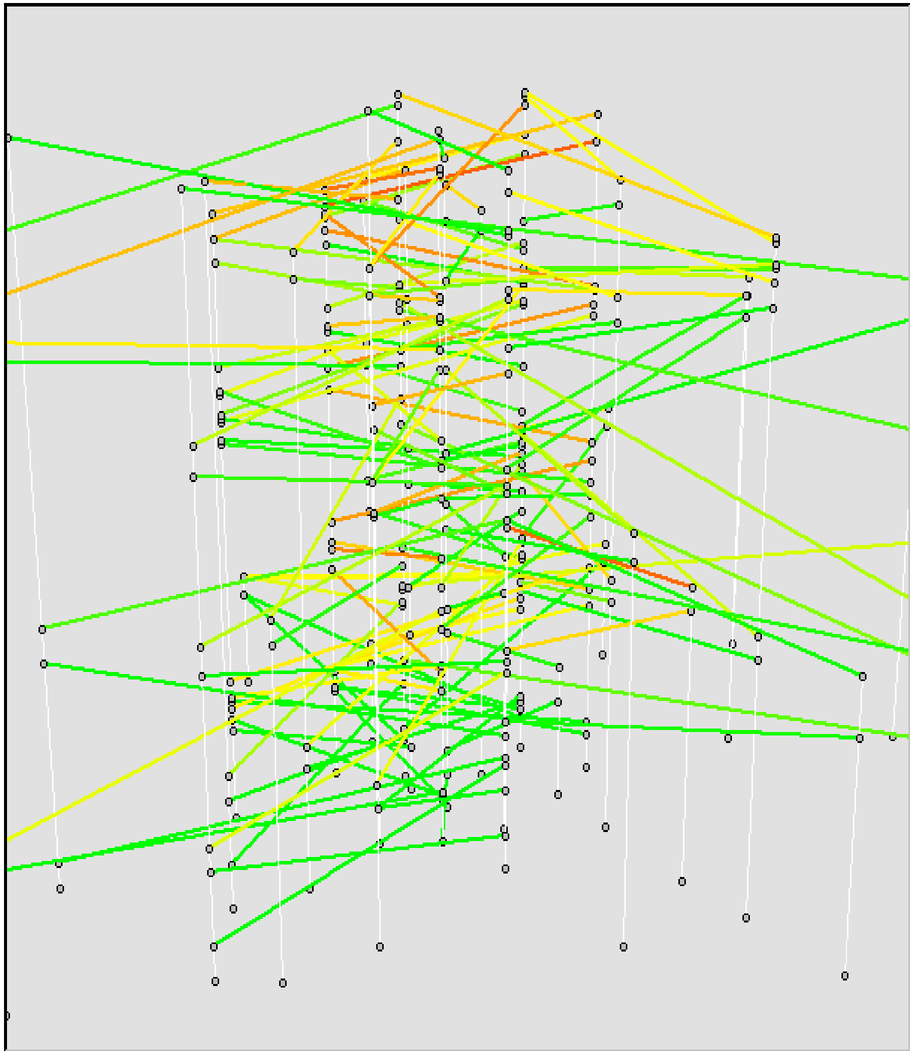
The Tail Assignment Problem – assign legs to aircraft in order to fulfill operational constraints such as preassignments, maintenance rules, airport curfews, and minimum connection times between legs, cf. Grönkvist [2005]



EDP (bad)



EDP (good)



Goal: Decrease impact of delays

- Primary delays: genuine disruptions, unavoidable
- Propagated delays: consequences of aircraft routing, can be minimized

Rule-oriented planning

- Ad-hoc formulas for buffers
- These rules are costly and it is uncertain how efficient they are
- Calibrating these rules is a balancing act: supporting operational stability, while staying cost efficient

Goal-oriented planning

- Minimize occurrence of delay propagation on average

Delay distribution

- Delays are not homogeneously spread in the network
- Stochastic model must captures properties of individual airports and legs

Structure of the stochastic model

- Gate phase, representing time spent on the ground
- Flight phase, representing time spent en-route

Phase durations are modelled by probability distribution

- G_j is random variable for delay of gate phase of leg j
- F_j is random variable for duration of flight phase of leg j

Mathematical model:

$$\min \sum_{k} \sum_{r \in R_k} d_r x_r^k$$
▷ Minimize non-robustness

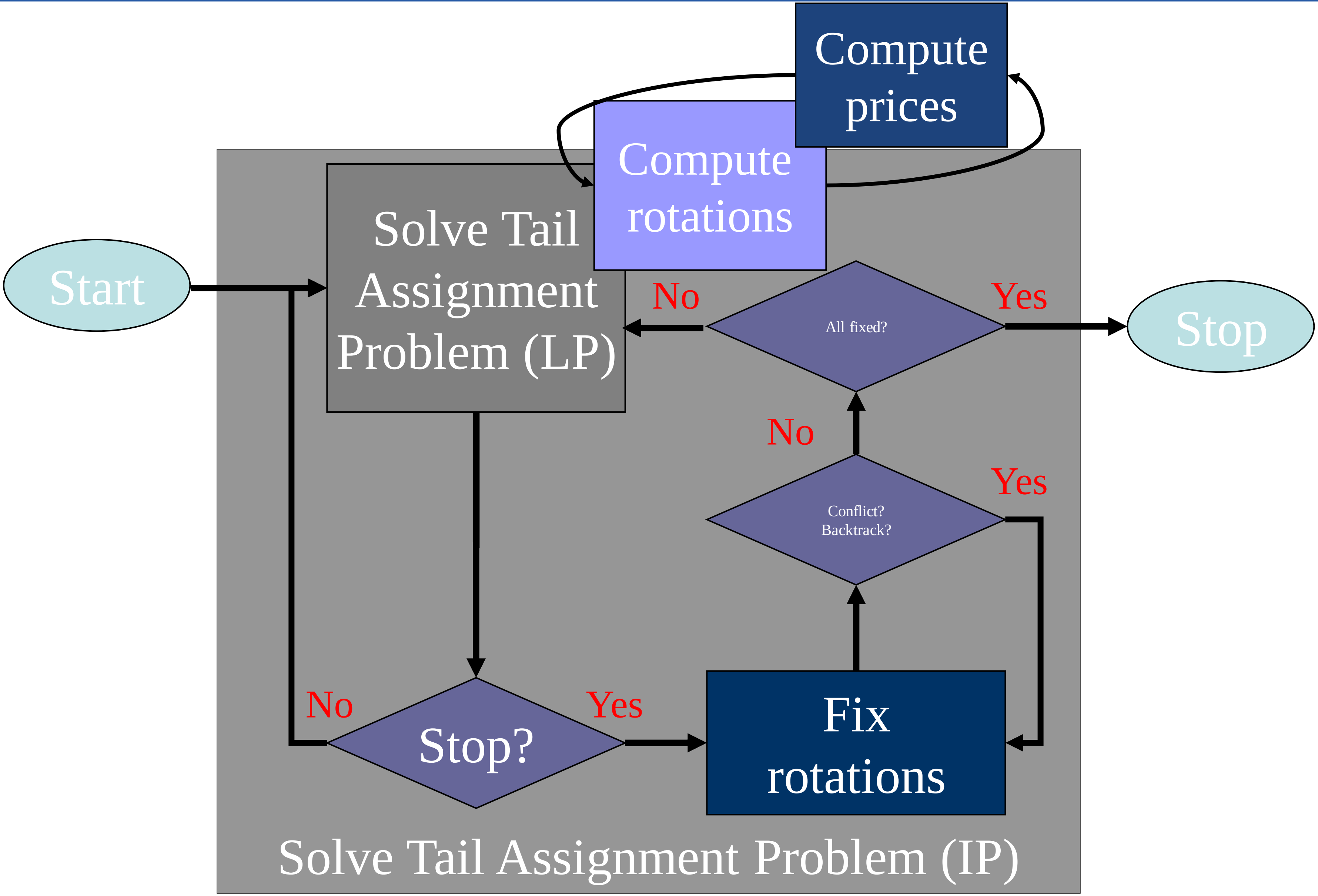
$$\sum_{k} \sum_{r: l \in r,\ r \in R_k} x_r = 1 \quad \forall l \in L$$
▷ Cover all legs

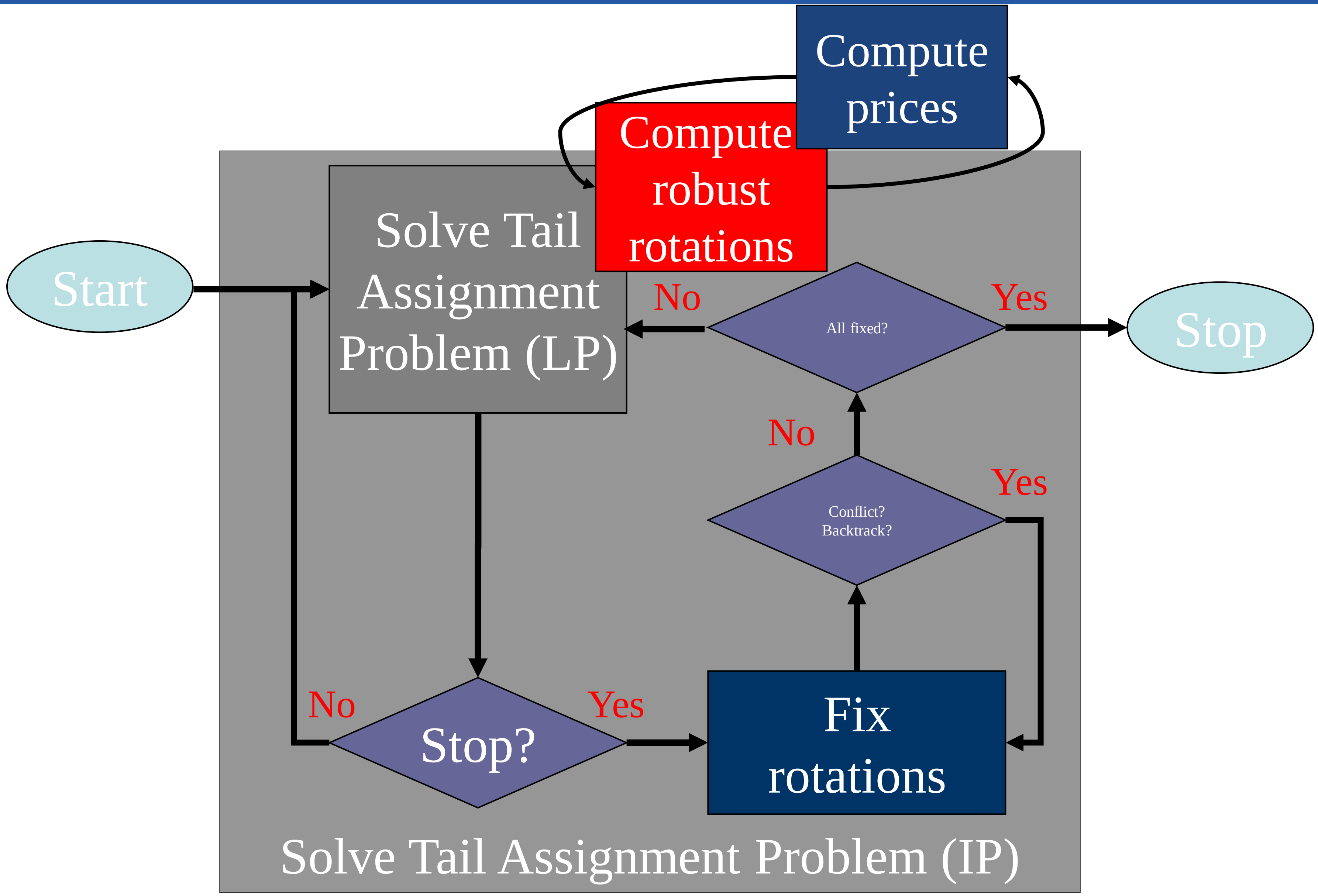
$$\sum_{k} \sum_{p \in R_k} a_{bp} x_p^k \le r_b \quad \forall b \in B$$
▷ Fulfill side constraints

$$\sum_{j \in R_k} x_j^k = 1 \quad \forall k$$
▷ One rotation for each aircraft

$$x_r^k \in \{0,1\} \quad \forall k, \forall r \in R_k$$
▷ Integrality

- Set partitioning problem with side constraints
- Problem has to be resolved daily for period of a few days
- Solved by Netline/Ops Tail xOPT (state-of-the-art column generation solver by Lufthansa Systems)


Compute prices
Compute rotations
Start
Solve Tail Assignment Problem (LP)
No
All fixed?
Yes
Stop
No
Yes
Conflict? Backtrack?
No
Stop?
Yes
Fix rotations
Solve Tail Assignment Problem (IP)


Compute
prices
Compute
robust
rotations
Start
Solve Tail
Assignment
Problem (LP)
No
All fixed?
Yes
Stop
No
Conflict?
Backtrack?
Yes
No
Stop?
Yes
Fix
rotations
Solve Tail Assignment Problem (IP)

- Robustness measure: total probability of delay propagation (PDP)

$$d_r = \sum_{i \in r} \mathrm{P}\left[PD_i^r > 0\right]$$

- Resource constraint shortest path problem

$$\min_{r \in R^k} \quad d_r - \sum_{i \in r} \pi_i + \sum_{b \in B} a_{br} \mu_b - \upsilon_k$$

where PD_i^r is random variable of delay propagated to leg i in rotation r and π_i, υ_k, μ_b are dual variables corresponding to cover, aircraft, and side constraints

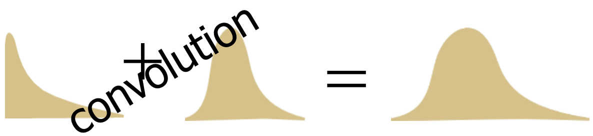
$$\min_{r \in R^k} \quad \sum_{i \in r} \mathrm{P}\left[PD_i^r > 0\right] - \sum_{i \in r} \pi_i + \sum_{b \in B} a_{br} \mu_b - \upsilon_k$$

To solve this problem one must compute PD_i^r along rotations

Computing PD_i Along a Rotation

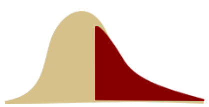
Delay distribution H_j of leg j

- $H_j = G_j + F_j$



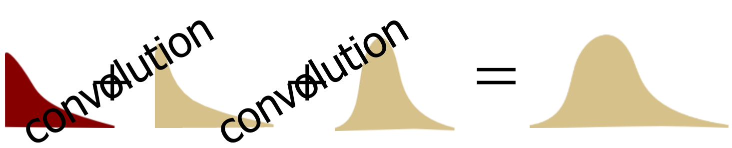
Delay propagation from leg j to leg k via buffer b_{jk}

- $PD_k = \max(H_j - b_{jk}, 0)$



Delay distribution H_k of next leg k

- $H_k = PD_k + G_k + F_k$

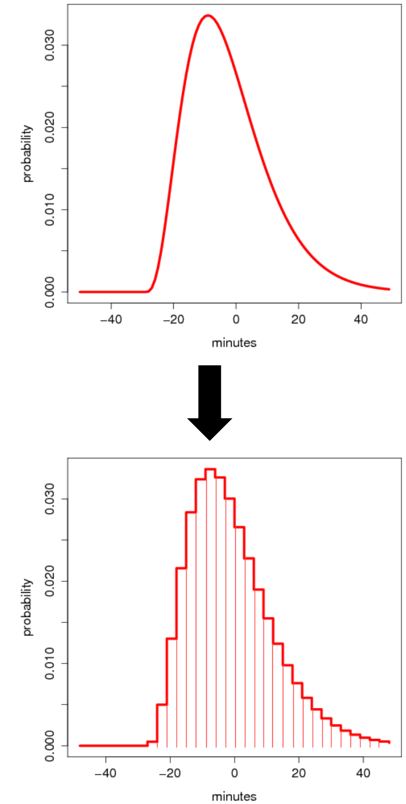


and so on…

Convolution

- ▷ H = F + G and f, g and h are their probability density functions

$$h(t) = \int_0^t f(x)g(t-x)dx$$



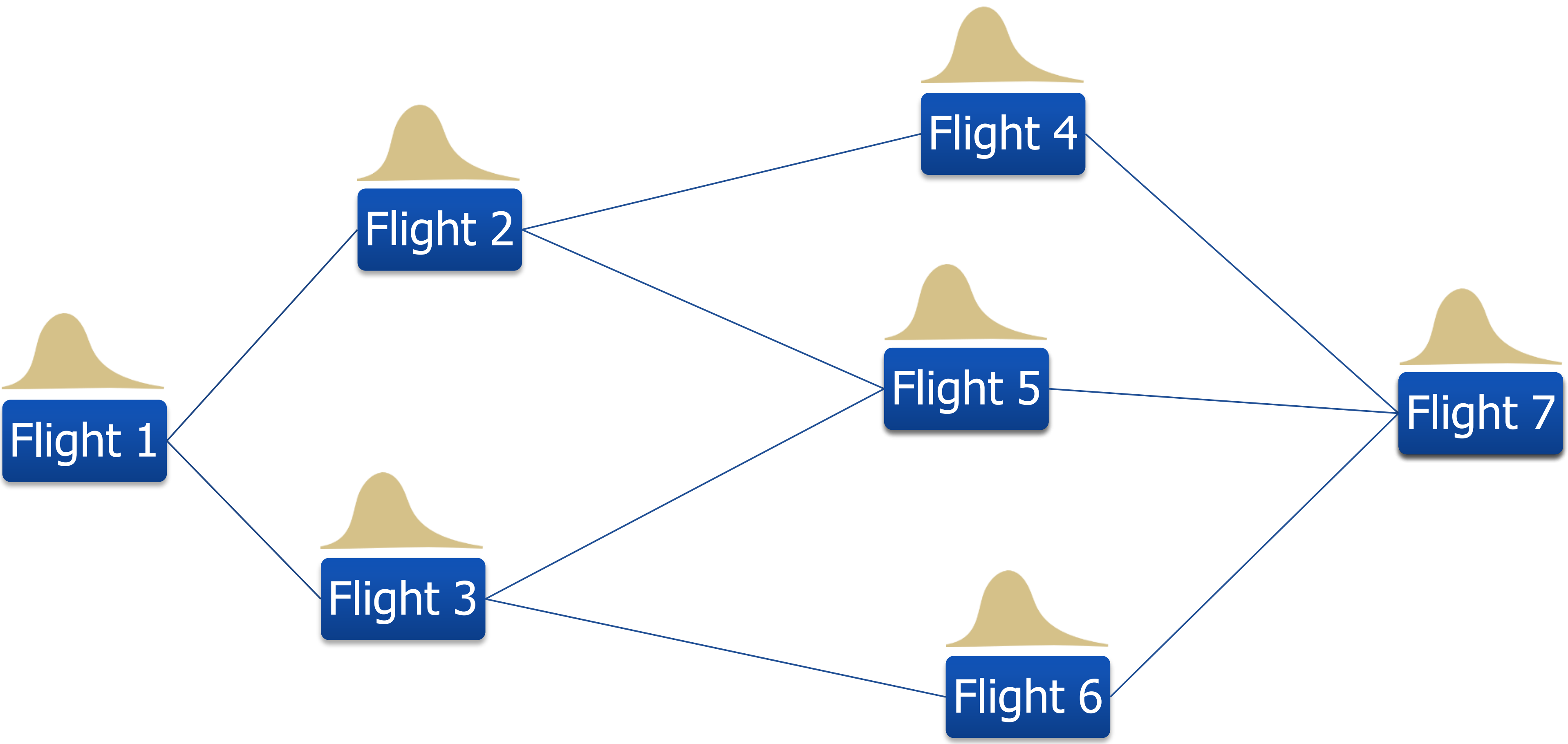
Numerical convolution based on discretization

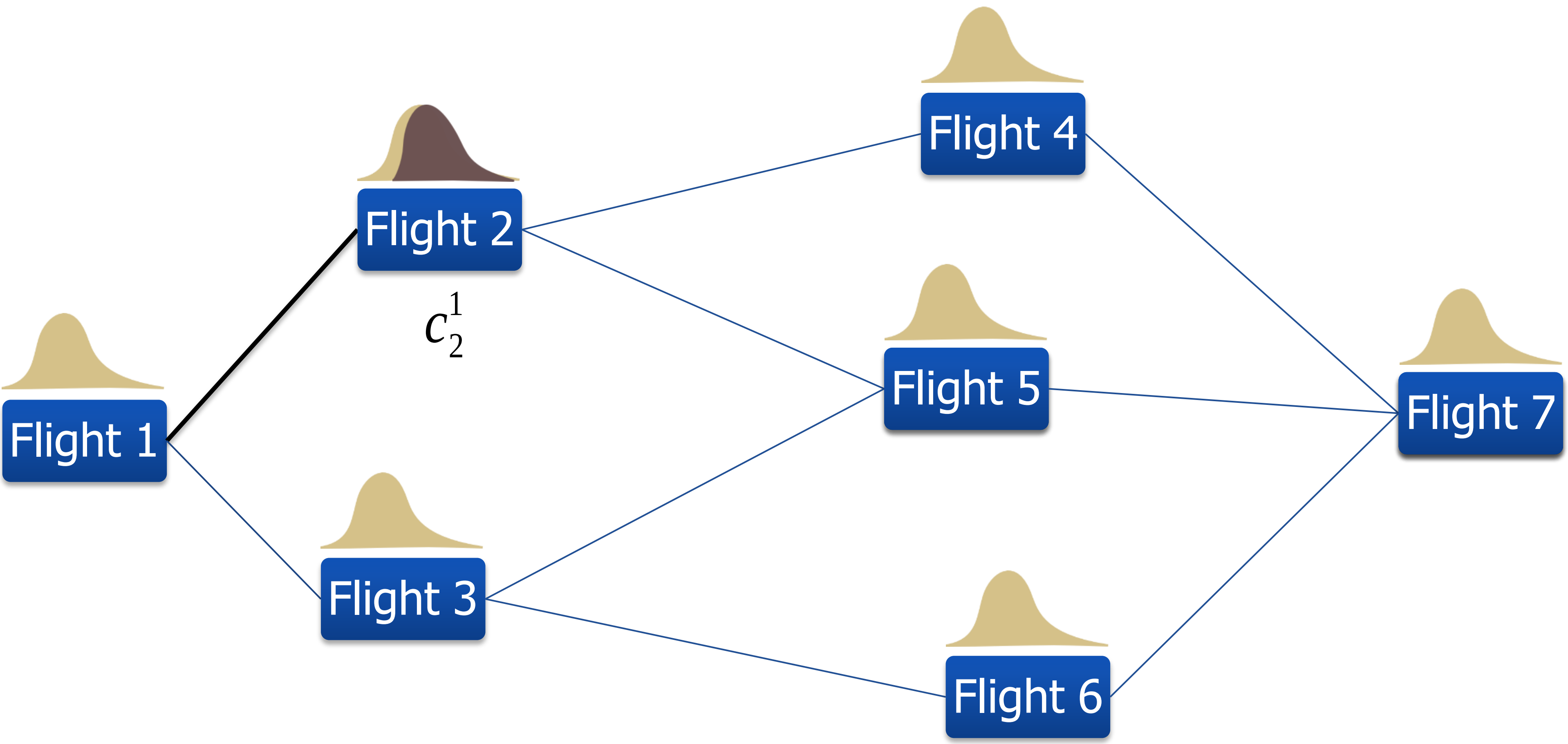
$$\overline{h}_t = \sum_{i=1}^{t} \overline{f}_i(\overline{g}_{t-i} + \overline{g}_{t-i-1})/2$$

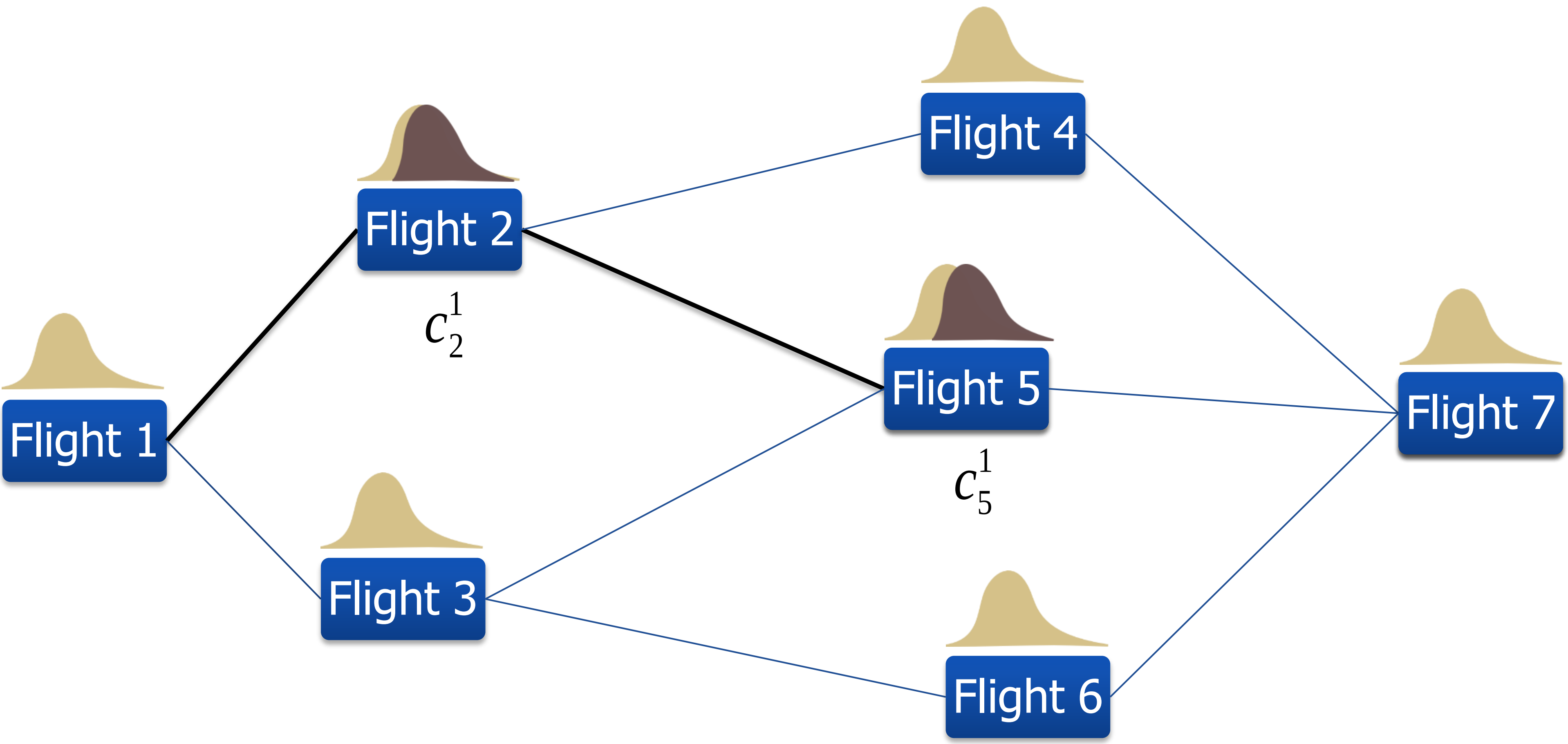
where $\overline{f}, \overline{g}$ are stepwise constant approximations of functions *f, g*

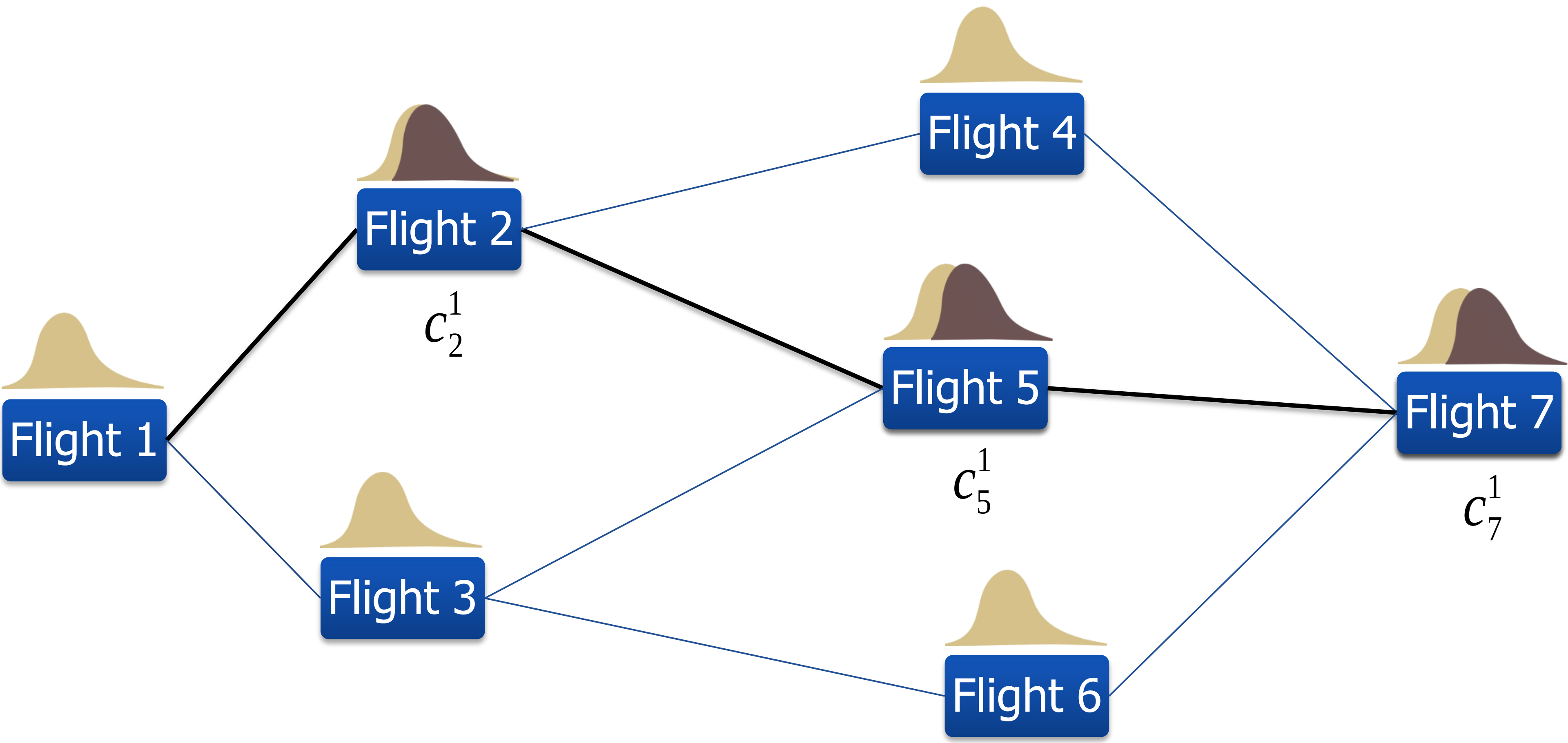
Alternative approaches

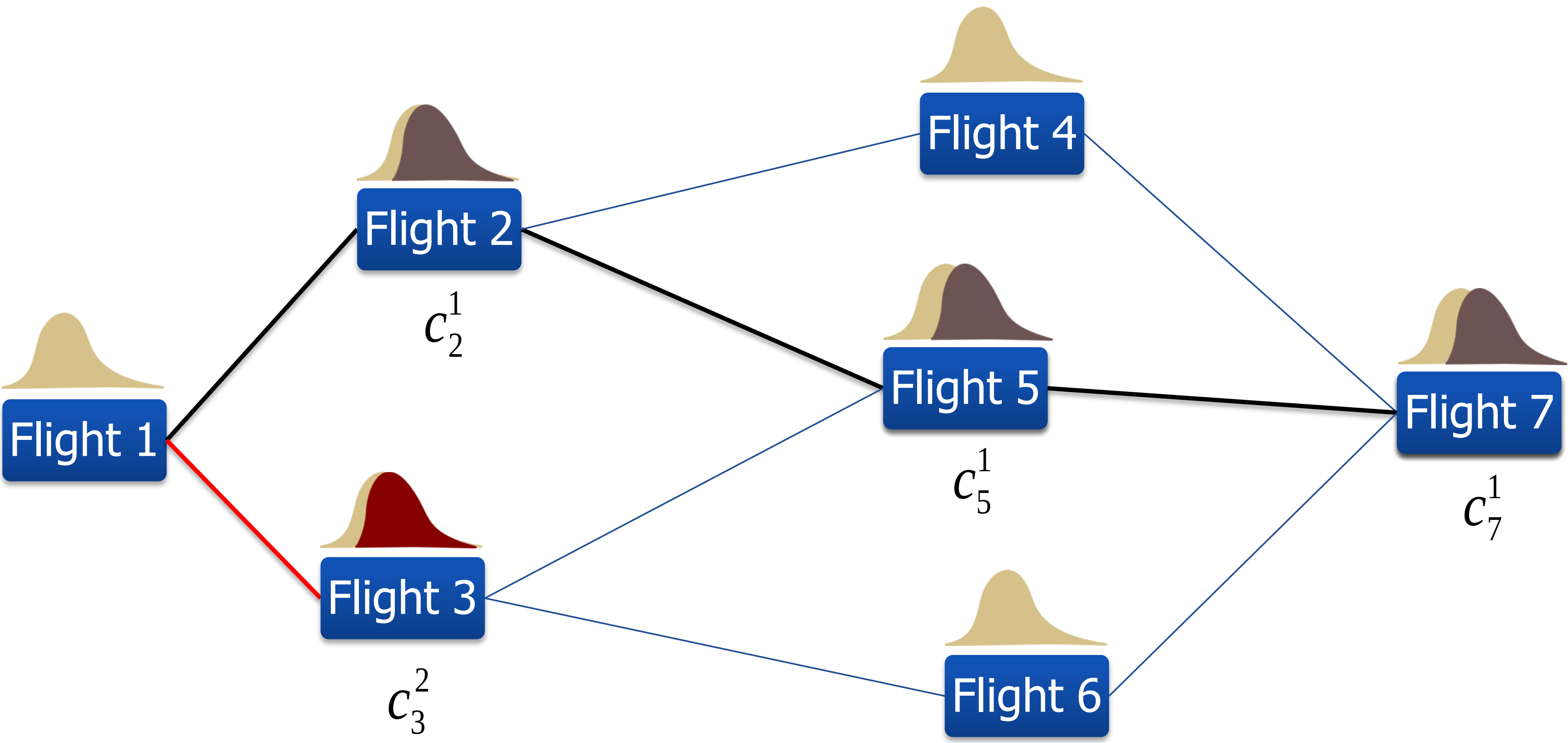
- ▷ Analytical convolution, cf. Fuhr [2007]

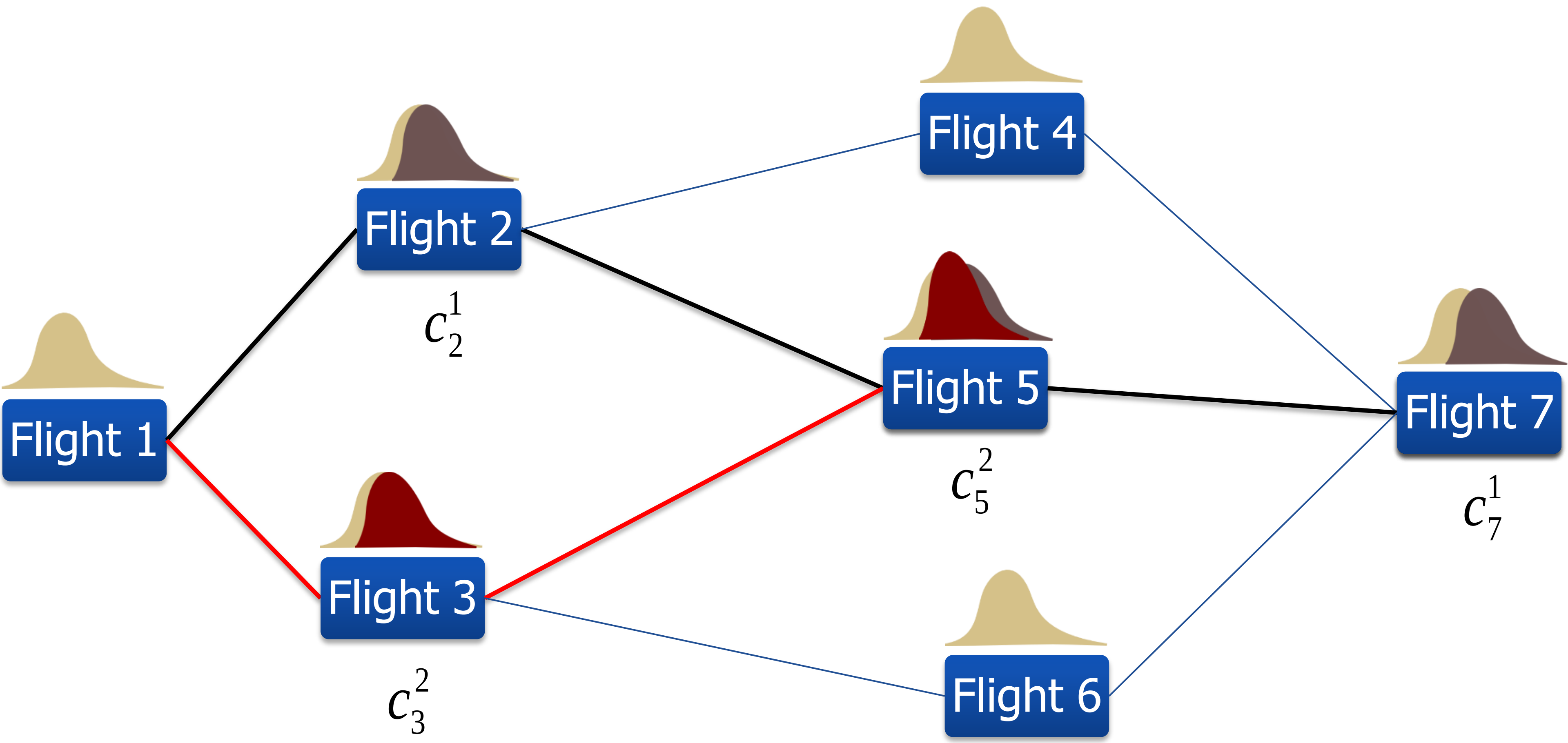

Flight 1
Flight 2
Flight 3
Flight 4
Flight 5
Flight 6
Flight 7

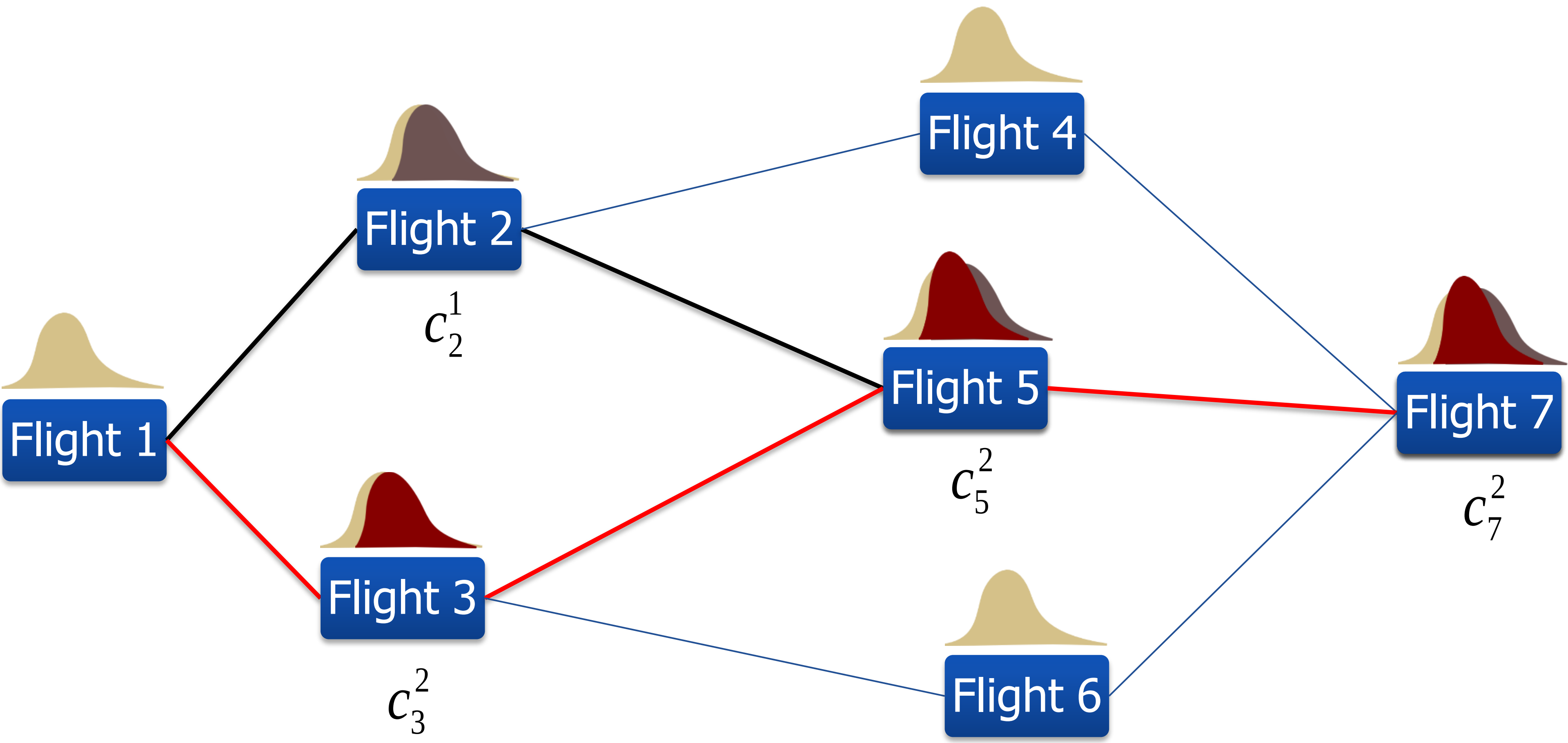

Flight 1
Flight 2
Flight 3
Flight 4
Flight 5
Flight 6
Flight 7
c_2^1


Flight 1
Flight 2
Flight 3
Flight 4
Flight 5
Flight 6
Flight 7
c_2^1
c_5^1


Flight 1
Flight 2
Flight 3
Flight 4
Flight 5
Flight 6
Flight 7
c_2^1
c_5^1
c_7^1


Flight 1
Flight 2
Flight 3
Flight 4
Flight 5
Flight 6
Flight 7
c_2^1
c_3^2
c_5^1
c_7^1


Flight 1
Flight 2
Flight 3
Flight 4
Flight 5
Flight 6
Flight 7
c_2^1
c_3^2
c_5^2
c_7^1


Flight 1
Flight 2
Flight 3
Flight 4
Flight 5
Flight 6
Flight 7
c_2^1
c_3^2
c_5^2
c_7^2

Instance SC1: reference solution

- 100 legs, 16 aircraft, no preassignments, no maintenace
- Optimizer produces the same solution for each step size
- CPU time differs only in computation of the convolutions
- PDP values differ because of approximation error

	step size [min]	CPU [s]	PDP	error [%]
SC1	0.1	15.4	25.0586	0.11
SC1	0.5	1.0	25.0672	0.15
SC1	1	0.5	25.0917	0.25
SC1	2	0.4	25.2227	0.77
SC1	3	0.4	25.4775	1.79
SC1	4	0.3	25.7667	2.94
Simulation*			25.0303	

Instance SC1: optimized solution

- Different discretization step sizes may produce different solutions
- CPU time and PDP are not straightforward to compare

	step size [min]	PDP optimized	CPU [s]	PDP simulated*
SC1	0.1	19.7268	4450	19.7469
SC1	0.5	19.7362	231	19.7382
SC1	1	19.7450	70	19.7239
SC1	2	19.8693	45	19.7313
SC1	3	20.0651	29	19.7239
SC1	4	20.3353	31	19.7562

Test Instances

Analyzed data

- approx. 350000 flights / 300 – 650 flights per day
- 28 months, 4 subfleets
- European airline with hub-and-spoke network

Test instances

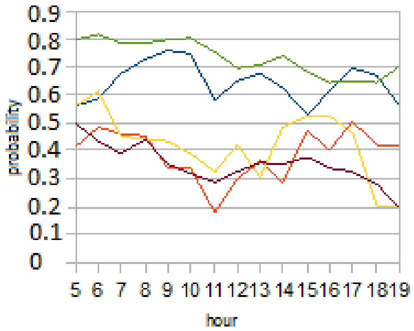
- We optimize single day instances of one subfleet
- Data for 4 months, no maintenance rules and preassignments

		min			**max**			**avg**		
	#days	Legs	aircraft	flight time [min]	legs	aircraft	flight time [min]	legs	aircraft	flight time [min]
January	26	44	12	3840	105	17	8830	88	15	7447
February	22	94	15	8295	118	17	10065	109	16	9339
March	21	94	15	7900	121	17	10390	110	16,3	9483
April	27	93	15	7080	118	18	9750	103	16	8648

Gate Phase

Probability of delay

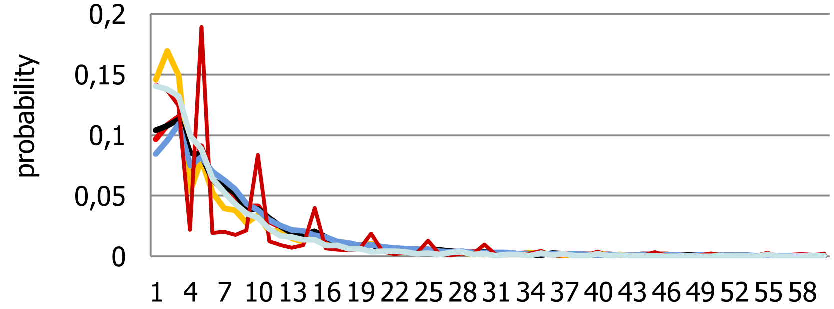
- Depends on day time and departure airport



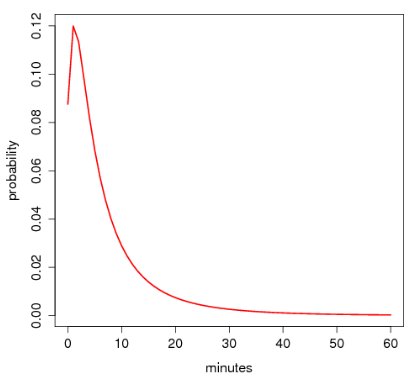
probability of departure delay during the day on various airports

Distribution of delay

- Independent of daytime and departure airport



distribution of the length of gate primary delays on various airports



Gate phase

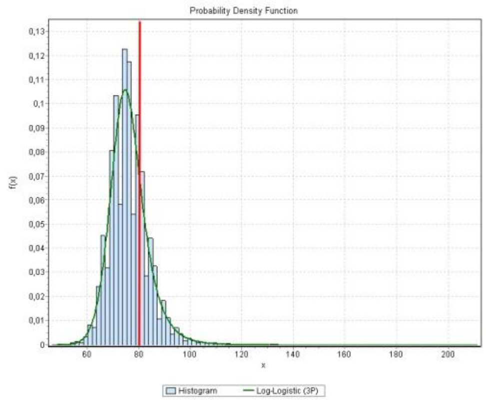
- gate delay distribution G_j of flight j

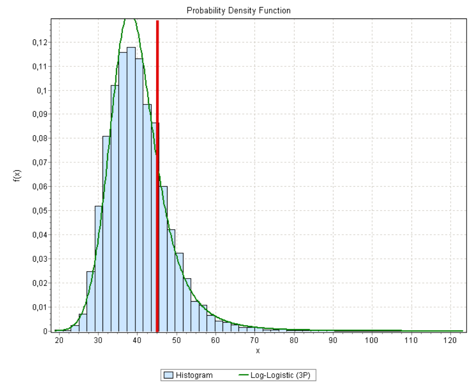
$$\Pr[G_j = x] = \begin{cases} 1 - p_j & x = 0 \\ p_j \operatorname{Ln}(x, \mu, \sigma) & x > 0 \end{cases}$$

where Ln() is probability density function of Log-normal distribution with Power-law distributed tail and $p_j = c(t(j), a(j))$, t(j) is departure time of flight j and a(j) is departure airport of flight j

Distribution of deviation from scheduled duration

- Depends on scheduled leg duration





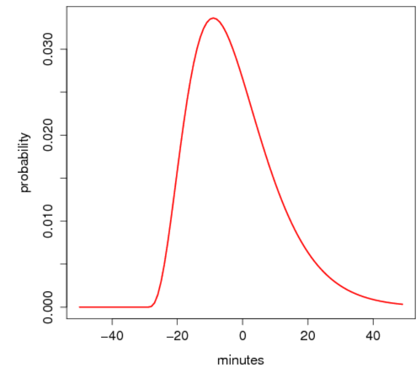
Histogram of the flight duration and its representation by random variable. left: scheduled flight duration 80 minutes, right: scheduled flight duration 45 minutes

Flight phase

- flight delay distribution F_j of flight j

$$\Pr[F_j = x] = \mathrm{Llg}(x + l_j, \alpha_{l_j}, \beta_{l_j}) \qquad x \in R$$

where Llg() is probability density function of Log-logistic distribution and l_j is scheduled flight duration of leg j



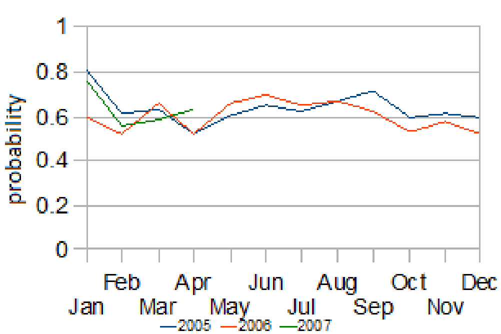
Parameters of the model:

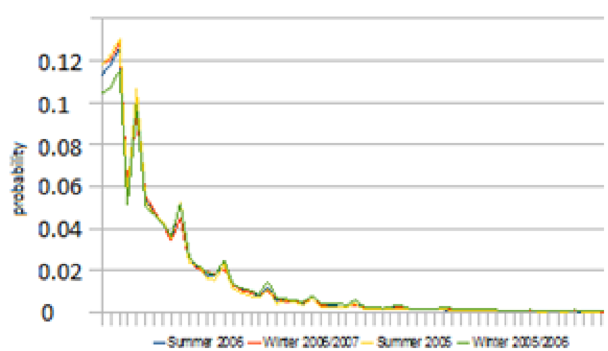
- p for every airport and day hour
- μ, σ
- α, β for every flight length
- Parameters are estimated by automatic scripts in R and quality is proofed by Chi-Square test.

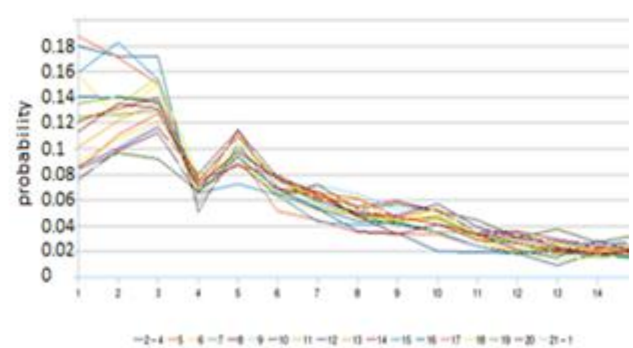
Model applied to South American airline data

Validation of various assumptions of the model

- Stability of parameters over time, …







ORC

- Standard KPI method
- Bonus for ground buffer minutes
- Threshold value for maximal ground buffer time (15 minutes)

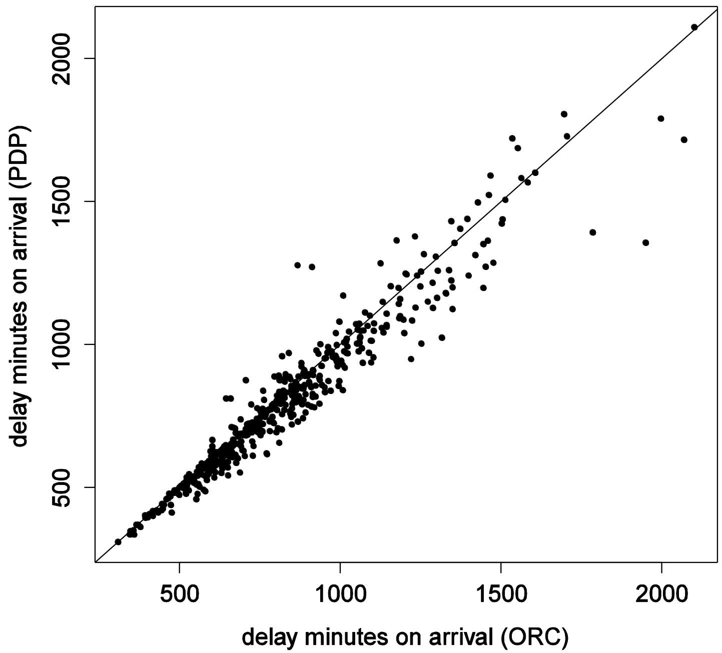
PDP

- Total probability of delay propagation

		ORC			PDP			Savings	
	#days	PDP	EAD [min]	CPU [s]	PDP	EAD [min]	CPU [s]	PDP	EAD [min]
January	26	414,51	28488	28	395,46	28085	66	19,05	403
February	22	540,48	31870	31	530,42	31652	89	10,06	218
March	21	516,69	30363	31	507,91	30174	75	8,78	189
April	27	465,48	34453	42	449,16	34159	71	16,51	294

ORC vs. PDP on a single disruption scenario

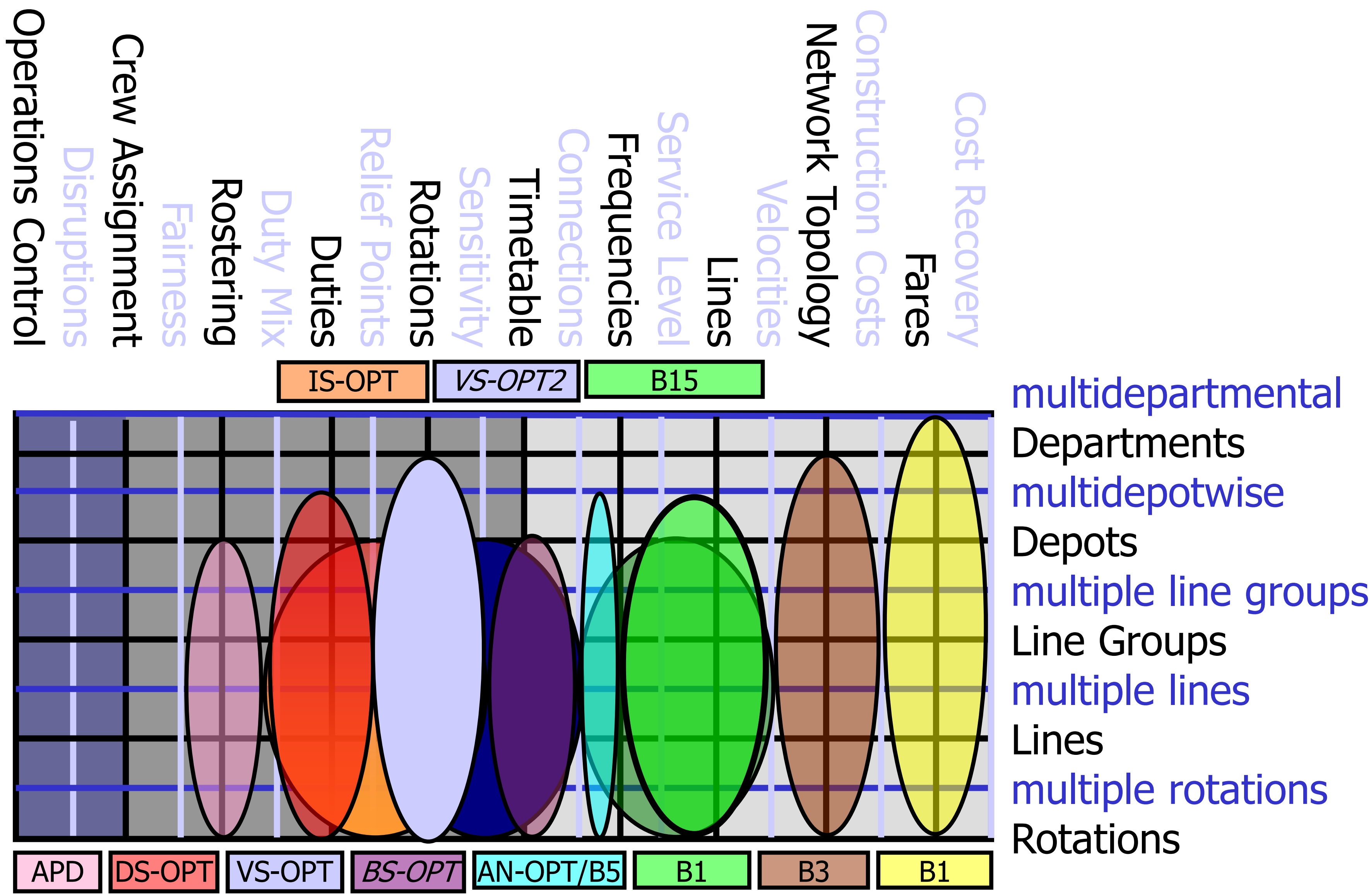
- ORC outperforms PDP only in 21% of cases
- PDP saves on average 29 minutes of arrival delay
- For more disrupted days, PDP saves on average 62 minutes of arrival delay



Estimation of monetary savings by the cost model developed based on EUROCONTROL [2004]

Lufthansa Systems estimates annual saving of the method in the tail assignment to 300,000 € for short haul carrier with 30 aircraft

Application in other planning stages may increase the benefit


Operations Control
Disruptions
Crew Assignment
Fairness
Rostering
Duty Mix
Duties
Relief Points
Rotations
Sensitivity
Timetable
Connections
Frequencies
Service Level
Lines
Velocities
Network Topology
Construction Costs
Fares
Cost Recovery
IS-OPT
VS-OPT2
B15
multidepartmental
Departments
multidepotwise
Depots
multiple line groups
Line Groups
multiple lines
Lines
multiple rotations
Rotations
APD
DS-OPT
VS-OPT
BS-OPT
AN-OPT/B5
B1
B3
B1

Visit ISMP 2012!

Thank your for your attention

PD Dr. habil. Ralf Borndörfer

Zuse-Institute Berlin
Takustr. 7
14195 Berlin-Dahlem

Fon (+49 30) 84185-243
Fax (+49 30) 84185-269
borndoerfer@zib.de

www.zib.de/borndoerfer