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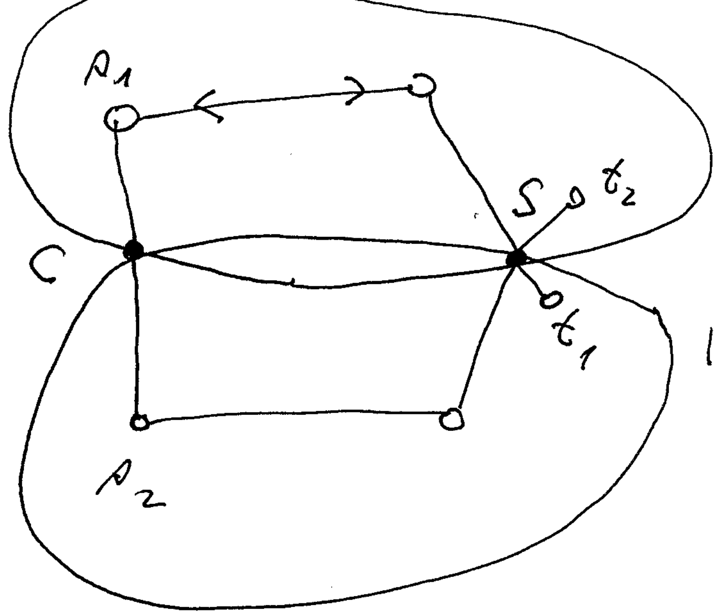
Prisoners' dilemma

P_1 \ P_2	Confess	Silent
Conf.	4, 4	1, 5
Silent	5, 1	2, 2

The only stable solution is <u>confess</u> BUT the best outcome is (sil., sil.)

Internet Service Providers routing

ISP1



Two networks exchange traffic via 2 peering points C, S
origin-destination points s_i t_i

$\boxed{\text{routing through C cheaper but hurts the other player}}$

Pollution Game

n countries

- polution control costs 3
- each poluting country adds 1 to costs of all countries

k (BAD) countries not control $\Rightarrow$

BAD country cost : k

GOOD country cost : $k+3$

Stable solution : no control, cost n

Good solution : cost 3 each

$\boxed{\text{Tragedy}}$ If $\sum x_i = 1/2 \Rightarrow$ total value is $1/4$

Tragedy of the commons

n players to share bandwidth (total capacity 1)

* quality deteriorates with total bandwidth used

Value of player i : $x_i(1 - \sum x_j)$

Strategy of i :

$t = \sum_{j \neq i} x_j < 1$. sending x.

gets profit $x(1-t-x)$:

$\boxed{\text{optimal is } x_i = (1-t)/2}$.

$\boxed{\text{Stable}}$ $(\forall i)\left(x_i = (1 - \sum_{j \neq i} x_j)/2\right)$

$\boxed{\text{unique solution}}$ $x_i = 1/(n+1)$

value : $1/(n+1)^2$, $\boxed{\text{total value} \sim \frac{1}{n}}$

Overusing common resource

Simultaneous Move Game

n players

S_i : set of strategies of player i

$s = (s_1, \dots, s_n)$ vector of strategies ; S set of all of them

Preference of outcome :

$u_i : S \to R$ (utility) $\quad c_i : S \to R$ (cost) $\quad u_i(s) = -c_i(s)$

<u>standard form of a game</u> : list all strategies and utilities

Typically large, COMACT ?

<u>dominant strategy</u> : $s \in S$ <u>dominant</u> if for each player i, $\forall s' \in S$,

$$\boxed{u_i(s_i, s'_{-i}) \geq u_i(s'_i, s'_{-i})}$$

$\boxed{Ex}$ Prisoners' dilemma

$\boxed{\text{Vickrey Auction}}$ [second price auction] $\boxed{\text{Mechanism design}}$

Need to DESIGN a game for auction a painting.

<u>Player i</u> : value v_i | payoff
- NO $\Rightarrow 0$
- YES $\Rightarrow v_i - p$

<u>Strategy:</u> the bid of player i

"I want to bid what the 2nd highest offer is + ε"

Deciding strategy (the bid) is hard problem, no dominant strategy if auction straightforward.

$\boxed{\text{Vickrey's mechanism}}$
- award painting to highest bid
- pay 2nd highest bid

$\boxed{\text{Each player's dominant strategy is to report true value as a bid}}$

(a) painting to player who values it most
(b) easy strategy not depend on other players

(Pure) Strategy Nash Equilibrium

$s \in S$ is Nash equilibrium if for all i and $\boxed{s_i' \in S_i}$

$$\boxed{u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})}$$

- Each dominant strategy is N.E.
- different N.O. may exist with different payoffs

Mixed Strategy N.E. | Random selection of strategies

<u>Mixed Strategy</u>: probability distribution over S_i

<u>value</u>: expected payoff

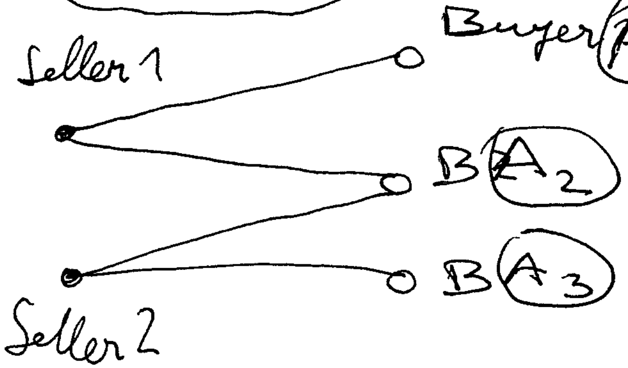
Thm (Nash) Any game with a finite set of players and finite set of strategies has a Nash Equilibrium of mixed strategies.

Finding Nash Eq: PPAD-complete problem

to find eq. we know exists

No N.E.

Pricing Game



No product cost

Each B : • buy 1 unit of product
• budget 1 each

Sellers : each names price $p_i \leq 1$

B buys from cheaper seller

(a) set $p_i = 1$ guarantee buyer 1, 3

(b) compete for B [∞ # strategies]

- $p_1 > \frac{1}{2} \Rightarrow 1/2 < p_2 < p_1$ profit > 1
- $p_1 \leq \frac{1}{2} \Rightarrow p_2 := 1 \Rightarrow p_1$ increases

Correlated Equilibrium

Traffic light

1 \ 2	cross	stop
c	-10, -10	1, 0
s	0, 1	0, 0

3 NEs $_0^{\,1}$, $_1^{\,0}$ and mixed:
cross with probability $1/11$.

The 1st two not fair,
The last one small payoff and risk.

coordinator chooses strategy for both players. BUT: need to be stable: each player find it in her interest to follow the recomended strategy.

(*) Coord. can randomly let 1 player cross. stopping player: payoff 0 but knows that crossing will cause accident.

Trusted Game Coordinator