Discrete & Computational Geometry manuscript No.
(will be inserted by the editor)

Erratum to: Improved Enumeration of Simple Topological Graphs

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Published online: date

1. Lemma 13 and the caption to Fig. 7 are false in the original version; the forbidden rotation should be $(3,2,1,4,5,6)$. Here is a correct version of Lemma 13.

Lemma 13 *Let G be a simple complete topological graph with vertices* $1,2,\ldots,7$. *Suppose that G contains a twisted graph* T_6 *induced by the vertices* $1,2,\ldots,6$*, in this canonical order, and with the orientation where the rotation of the vertex* 6 *is* $(1,2,3,4,5)$. *Then the rotation of the vertex* 7 *is not* $(3,2,1,4,5,6)$.

Proof. Suppose for contradiction that the rotation of the vertex 7 is $(3,2,1,4,5,6)$. The subgraphs $G_1 = G[\{1,2,3,4\}]$ and $G_2 = G[\{3,4,5,6\}]$ are both isomorphic to the convex graph C_4. The 4-cycles corresponding to the outer face of C_4 are 1243 and 3465, respectively. The two triangular faces adjacent to the vertices 3 and 4 in G_1 and G_2 cover the whole plane; see Figure 7. It follows that at least one of these two faces contains the vertex 7. The rotation of the vertex 7 is $(1,4,3,2)$ in $G[1,2,3,4,7]$ and $(3,4,5,6)$ in $G[3,4,5,6,7]$, which contradicts Lemma 12. □

2. In Subsection 3.6, it is claimed that "Each bounded face of G' is an intersection of the interiors of a particular subset of triangles of G'". This is not true already for some drawings of K_4. Moreover, when defining the equivalence of faces, one should also take the orientation of the drawings into account. The two sentences starting with "Each bounded face …" on the last line of page 742 should be replaced as follows.

Two faces F_1' and F_2' in two simple complete topological graphs G_1' and G_2' weakly isomorphic to G' are considered *equivalent* if every triangle T_1 in G_1' and the corresponding triangle T_2 in G_2' satisfy the following condition: the triangles T_1 and T_2 have the same orientation if and only if either T_1 contains F_1' and T_2 contains F_2', or F_1' is outside T_1 and F_2' is outside T_2.

In addition, the last sentence in the first paragraph on page 743 should be replaced as follows.

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Therefore, there are at most 2^{n-1} possible sets $R(f)$. Accounting for two possible orientations of the drawing of C_n, we get the upper bound $f(C_n) \leq 2^n + 2$.

3. There is a typo in the statement of Lemma 20: the "(F)" should be "$f(F)$".

4. The remark after Proposition 6 about extension to the wheel graph W_4 or even $K_5 - K_2$ is false, see e.g. Figure 8 in [1].

References

1. M. Schaefer, Taking a detour; or, Gioan's theorem, and pseudolinear drawings of complete graphs, *Discrete & Computational Geometry* **66** (2021), 12–31.