Exercises for Combinatorial and Computational Geometry 2

Series 1 — Erdős–Szekeres theorem

hints 5. 3. 2025, deadline 11. 3. 2025

Please choose a nickname that will be used in the list of scores on the webpage of the exercises. If you have already chosen a nickname, you can just sign your solutions with either your name or your nickname.

1. (a) Show that for every $k \in \mathbb{N}$ there is an $n(k) \in \mathbb{N}$ such that every set of $n(k)$ points in the plane contains k points in *general position* or k points on a common line. **[2]**

   (b) Prove the previous statement with $n(k)$ at most polynomial in k. **[1]**

2. (a) Prove the Erdős–Szekeres theorem in $\mathbb{R}^d$: for every $d \geq 3$ and $k \in \mathbb{N}$ there is an $n = n_d(k) \in \mathbb{N}$ such that every set of n points in $\mathbb{R}^d$ in *general position* (no $m+2$ points in an affine subspace of dimension m, for $m = 1, 2, \ldots, d-1$) contains a subset of k points in convex position. **[2]**

   (b) Show that every sufficiently large finite point set in $\mathbb{R}^3$ in general position contains a 7-hole. **[2]**

3. Let P be a set of $3n-1$ points in the plane in convex position. Every closed segment between two points in P is colored either red or blue. Prove that there exist n pairwise disjoint red segments or n pairwise disjoint blue segments. **[3]**

4. Prove that there is a sufficiently large constant C such that the $n \times n$ grid $\{(i,j);\ i = 1, 2, \ldots, n;\ j = 1, 2, \ldots, n\}$ has no subset in convex position with more than $Cn^{2/3}$ points. **[4, hint]**

5. Prove that for $h \geq 1$, the Horton set with 2^h points has no subset in convex position with more than $4h$ points. **[2]**

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web: `https://kam.mff.cuni.cz/~kvgweb/kvg/eng.html`