

# Exercises for Combinatorial and Computational Geometry

## Series 1 — Convex sets

hints 20.10.2021, deadline 15:40 27.10.2021

**Please choose a nickname that will be used in the list of scores on the webpage of the exercises. If you have already chosen a nickname, you can just sign your solutions with either your name or your nickname.**

1. Find a set  $M \subset \mathbb{R}^2$  that is a union of two convex sets such that  $\mathbb{R}^2 \setminus M$  consists of five pairwise disjoint connected components. [2]
2. Prove Carathéodory's theorem (you may use Radon's theorem or a part of its proof). [2]
3. Let  $M = \{x_1, y_1, x_2, y_2, \dots, x_{d+1}, y_{d+1}\}$  be a set of  $2d + 2$  points in  $\mathbb{R}^d$ . Prove that  $M$  can be partitioned into two subsets  $A$  and  $B$  such that each of these subsets contains, for every  $i = 1, 2, \dots, d + 1$ , exactly one point from  $\{x_i, y_i\}$ , and the convex hulls of  $A$  and  $B$  have a nonempty intersection. (You may use the fact that the  $(d + 1)$ -tuple of vectors  $x_i - y_i$  is linearly dependent and then use an approach similar to the proof of Radon's theorem.) [2]
4. Let  $M$  be a finite set of at least four points in the plane such that each point is either red or blue. In addition, for every 4-tuple  $V$  of points of  $M$  there is a line strictly separating the red points of  $V$  from the blue points of  $V$ . Prove that there is a line strictly separating all the red points of  $M$  from all the blue points of  $M$ . [3]
5. Let  $X_1, X_2, \dots, X_{d+1}$  be finite point sets in  $\mathbb{R}^d$  such that for every  $i \in \{1, 2, \dots, d + 1\}$  the origin lies in  $\text{conv}(X_i)$ . Prove that there exist  $d + 1$  points  $x_1, x_2, \dots, x_{d+1}$ , with  $x_i \in X_i$ , such that  $\text{conv}(\{x_1, x_2, \dots, x_{d+1}\})$  contains the origin. [4, hint]