

# Exercises for Combinatorial and Computational Geometry

## Series 1 — Convex sets

hints 27.10.2016, deadline 3.11.2016

Please choose a nickname that will be used in the list of scores on the webpage of the exercises. If you have already chosen a nickname, you can just sign your solutions with either your name or your nickname.

1. Find a set  $M \subset \mathbb{R}^2$  that is a union of two convex sets such that  $\mathbb{R}^2 \setminus M$  consists of five pairwise disjoint connected components. [2]
2. Prove Carathéodory's theorem (you may use Radon's theorem or a part of its proof). [2]
3. Let  $M = \{x_1, y_1, x_2, y_2, \dots, x_{d+1}, y_{d+1}\}$  be a set of  $2d+2$  points in  $\mathbb{R}^d$ . Prove that  $M$  can be partitioned into two subsets  $A$  and  $B$  such that each of these subsets contains exactly one point from  $\{x_i, y_i\}$  for every  $i = 1, 2, \dots, d+1$  and the convex hulls of  $A$  and  $B$  have a nonempty intersection. (You may use the fact that the  $(d+1)$ -tuple of vectors  $x_i - y_i$  is linearly dependent and then use an approach similar to the proof of Radon's theorem.) [2]
4. Let  $M$  be a finite set of at least four points in the plane such that each point is either red or blue. In addition, for every 4-tuple  $V$  of points of  $M$  there is a line which strictly separates red points of  $V$  from blue points of  $V$ . Prove that there is a line which strictly separates all red points of  $M$  from all blue points of  $M$ . [3]
5. Let  $X_1, X_2, \dots, X_{d+1}$  be finite point sets in  $\mathbb{R}^d$  such that for every  $i \in \{1, 2, \dots, d+1\}$  the origin lies in  $\text{conv}(X_i)$ . Prove that there exist  $(d+1)$  points  $x_i \in X_i$ ,  $i \in \{1, 2, \dots, d+1\}$ , such that the origin lies in  $\text{conv}(\{x_1, x_2, \dots, x_{d+1}\})$ . [4, hint]