Exercises for Combinatorial and Computational Geometry

Series 1 — Convex sets

hints 27.10.2015, deadline 3.11.2015

Please choose a nickname that will be used in the list of scores on the webpage of the exercises. If you have already chosen a nickname, you can just sign your solutions with either your name or your nickname.

1. Find a set $M \subset \mathbb{R}^2$ that is a union of two convex sets such that $\mathbb{R}^2 \setminus M$ consists of five pairwise disjoint connected components. **[2]**

2. Prove Carathéodory's theorem (you may use Radon's theorem or a part of its proof). **[2]**

3. Let $M = \{x_1, y_1, x_2, y_2, \ldots, x_{d+1}, y_{d+1}\}$ be a set of $2d+2$ points in $\mathbb{R}^d$. Prove that M can be partitioned into two subsets A and B such that each of these subsets contains exactly one point from $\{x_i, y_i\}$ for every $i = 1, 2, \ldots, d+1$ and the convex hulls of A and B have a nonempty intersection. (You may use the fact that the $(d+1)$-tuple of vectors $x_i - y_i$ is linearly dependent and then use an approach similar to the proof of Radon's theorem.) **[2]**

4. We say that a set P *pierces the triangles* of a point set M if every triangle determined by a triple of points of M contains at least one point of P in its interior. Prove that for every n-point set $M \subset \mathbb{R}^2$ in general position (no three points on a line) there is a set P of $2n-5$ points that pierces the triangles of M. **[3]**

5. Let $X_1, X_2, \ldots, X_{d+1}$ be finite point sets in $\mathbb{R}^d$ such that for every $i \in \{1, 2, \ldots, d+1\}$ the origin lies in $\mathrm{conv}(X_i)$. Prove that there exist $(d+1)$ points $x_i \in X_i$, $i \in \{1, 2, \ldots, d+1\}$, such that the origin lies in $\mathrm{conv}(\{x_1, x_2, \ldots, x_{d+1}\})$. **[4, hint]**

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