

Full Name:

1	2	3	4	$\Sigma$

- 
- Write legibly,
  - Write the solution to each problem on the same sheet as the problem statement.
  - For every problem, justify your answer; a correct answer without an explanation is worthless.
  - No aids such as notes, calculators, or phones are permitted.
- 

1. What do you know about eigenvectors corresponding to distinct eigenvalues of a given matrix? State precisely and prove.

2. Decide whether the matrices  $C = \begin{pmatrix} 1 & 0 & 0 \\ -4 & -1 & -2 \\ 8 & 4 & 5 \end{pmatrix}$  and  $D = \begin{pmatrix} -7 & -4 & -8 \\ 4 & 3 & 4 \\ 8 & 4 & 9 \end{pmatrix}$  are similar. **15**

- 3.** a) Consider a vector space  $V$ , a vector  $v \in V$ , and let  $B = (b_1, \dots, b_n)$  be some orthonormal basis of it. State the theorem describing how to obtain the coordinates of a vector  $u$  with respect to the basis  $B$ . **5**
- b) Consider the vector space  $V = \mathbb{R}^3$  and let  $U$  be its subspace generated by the vector  $b_1 = (1, 1, 1)^T$ . Find a basis of the orthogonal complement of  $U$ , with respect to the standard inner product, such that any two of its vectors  $u, v, u \neq v$ , are orthogonal to each other. **10**

4. Which of the following statements are true? Justify your answers.

- (a) For every symmetric matrix  $A$ , if  $\det(A) > 0$ , then  $A$  is positive definite. **5**
- (b) For any two matrices  $A, B$  (of size  $n \times n$ ) over the complex numbers, the following holds: If  $\lambda_1, \dots, \lambda_n$  are eigenvalues of the matrix  $A$  and  $\lambda'_1, \dots, \lambda'_n$  are eigenvalues of the matrix  $B$  (each eigenvalue counted with its algebraic multiplicity), then  $\lambda_1 \cdot \lambda'_1, \dots, \lambda_n \cdot \lambda'_n$  are eigenvalues of the matrix  $(AB)$ . **5**
- (c) The quadratic form  $f(x) = x^T \begin{pmatrix} -2 & -1 & 0 & -3 \end{pmatrix} x$  on  $\mathbb{R}^2$  takes only negative values, except at the vector  $x = (0, 0)^T$ . **5**