Exercise 1: Prove that for orthogonal vectors $q_1, q_2, \ldots, q_n \in \mathbb{R}^n$ matrices $I - q_1 q_i^T, \ldots, I - q_n q_n^T$ commute with respect to matrix product.

Exercise 2: Prove that matrix $H = I - 2uu^T$ is orthogonal (that is $H^T H = I$).

Exercise 3: Let $(x_i)_{i=1}^{\infty}$ be a convergent series of vectors in V and let $x = \lim_{i\to\infty} x_i$ Prove that if $y \in V$ is perpendicular to all x_i's then it is perpendicular to x also.

Exercise 4: Find the distance of point $A = (5, 5, 3, 3)$ from a plane given by points $B = (8, -1, 1, -2), C = (4, -2, 2, -1)$ and containing the origin.

Exercise 5: Which of the following properties has the relation of orthogonality: reflexivity, irreflexibility, symmetry, antisymmetry, transitivity?

Exercise 6: Suppose A is an $n \times k$ matrix, where $k \leq n$, such that the columns of A are linearly independent. Then the $k \times k$ matrix $A^T A$ is invertible.

Exercise 7: Suppose that M is an $n \times n$ matrix such that $M^T = M = M^2$. Let W denote the comlumn space of M.

1. Suppose that $Y \in W$. Prove that $MY = Y$.
2. Suppose that v is a vector in $\mathbb{R}^n$. Why is $Mv \in W$?
3. If $Y \in W$, why is $v - Mv \perp Y$?
4. Conclude that Mv is the projection of v into W.

Exercise 8: Compute the projection of the vector $v = (1, 1, 0)$ onto the plane $x + y - z = 0$.