Exercise 1:[Corrected] Prove that $\frac{x}{2}+\frac{y}{3}+\frac{z}{6}\le\frac{x^2}{2}+\frac{y^2}{3}+\frac{z^2}{6}$ holds for $x,y,z\in\mathbb{R}$.

Exercise 2: For a matrix

$$A=\begin{pmatrix}1 & 2 & 1\\ 2 & 4 & 3\\ 3 & 6 & 4\end{pmatrix}$$

find a non-zero vector perpendicular

1. to all rows of A,
2. to all vectors in $\mathcal{R}(A)$,
3. to all vectors in $\mathcal{C}(A)$.

Exercise 3: Prove that $5a_1+a_2+3a_3+a_4\le 6\sqrt{a_1^2+a_2^2+a_3^2+a_4^2}$ holds for all $a_1,a_2,a_3,a_4\in\mathbb{R}$.

Exercise 4:[A-G inequality] Prove that

$$\frac{a_1+a_2+\cdots+a_n}{n}\le\sqrt{\frac{a_1^2+a_2^2+\cdots+a_n^2}{n}}$$

hold for every $a_1,\ldots,a_n\in\mathbb{R}$.

Exercise 5: Prove that $\|A\|:=\sqrt{\sum_{i=1}^{m}\sum_{j=1}^{1}a_{i,j}^2}$ is a norm in the space of real $m\times n$ matrices.

Exercise 6: Find orthonormal basis of a vecotr space generated by $x=(1,0,1,0), y=(1,1,1,1), z=(1,0,0,1)$.

Exercise 7: What happens during Gramm-Schmidt process when we input

1. linear dependent vectors,
2. orthogonal vectors,
3. orthonormal vectors,
4. $-x_i$ instead of x_i? Inspect the change in the output of the procedure.

Exercise 8: Let $x_1=(1,1,0), x_2=(1,1,1))$. Find the result of G-S process when the input is x_1,x_2 and when the input is x_2,x_1. Find a projection of vector $x=(0,1,1)$ onto space $U=\operatorname{span}\{x_1,x_2\}$.

Exercise 9: Run G-S process on vectors $(i,i,i),(0,i,i),(0,0,i)$ in $\mathbb{C}$.

Exercise 10: Use the projection to find the best approximate solution of the system $\mathbf{Ax}=\mathbf{b}$, where

$$\mathbf{A}=\begin{pmatrix}2 & 1 & 0\\ 4 & 2 & 0\\ 2 & -4 & -1\\ 1 & -2 & 2\end{pmatrix},\qquad \mathbf{b}=(10,5,13,9)^T$$

Observe that the columns of $\mathbf{A}$ are mutually perpendicular.