Exercise 1: Interpolate plane $ax + by + cz + d = 0$ through points

a) $(6, 4, 6)$, $(3, 5, 4)$ and $(5, 2, 3)$

b) $(5, 4, 7)$, $(4, 5, 5)$ and $(2, 2, 6)$

Exercise 2: Calculate a) $(3 + 2i)(1 - 3i)$

b) $(3 + 2i)/(1 - 3i)$

c) $(1 + i)^{20}$

Exercise 3: Interpolate a cubic polynomial through points $(-2, 5)$, $(-1, 2)$, $(1, -4)$, and $(2, 5)$.

Exercise 4: Show that a matrix $\mathbf{A}$ of order $m \times n$ has rank one if and only if it can be written as a product of two nonzero matrices: $\mathbf{B}$ of order $m \times 1$, and $\mathbf{C}$ of order $1 \times n$.

(Indeed $\mathbf{B}$ can be viewed as a vector and $\mathbf{C}$ as a transpose of some vector.)

Exercise 5: Solve the following matrix equation in the field $\mathbb{Z}_5$.

$$\mathbf{A} = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 2 & 0 & 1 & 1 \\ 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 & 2 & 2 & 1 \\ 1 & 0 & 2 & 0 \\ 2 & 1 & 0 & 2 \\ 2 & 2 & 1 & 1 \end{pmatrix}, \quad \mathbf{C} = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix},$$

$$\mathbf{D} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 2 & 1 \end{pmatrix}, \quad \mathbf{E} = \begin{pmatrix} 1 & 1 & 2 & 0 \\ 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 1 & 2 & 0 & 0 \end{pmatrix}$$

a) $\mathbf{A}^T(\mathbf{X} - 2\mathbf{D})^{-1} + \mathbf{C} = \mathbf{B} - 2\mathbf{E}$

b) $\mathbf{B}^T(\mathbf{X} - 2\mathbf{D})^{-1} + \mathbf{C} = 3\mathbf{A} - \mathbf{E}$

Exercise 6: Decide, whether the following sets of vectors are linearly independent in the space of real functions $\mathbb{R} \to \mathbb{R}$ (over the field $\mathbb{R}$).

a) $\{2x - 1, x - 2, 3x\}$.

b) $\{x^2 + 2x + 3, x + 1, x - 1\}$.

c) $\{\ln(x), \log(2x), \log_2(x^2)\}$.

(i.e. the natural, decadic and binary logarithm.)

Exercise 7: In the space of real polynomials of degree at most four with the basis $X = (x^4 + x^3,\ x^3 + x^2,\ x^2 + x,\ x + 1,\ x^4 + 1)$ determine coordinates $[f]_X$ of the following vectors f:

a) $f(x) = x^4 - 1$.

b) $f(x) = x^4 + x^3 + x^2 + x + 1$.

c) $f(x) = x^4 + x^2 + 1$.

d) $f(x) = x^3 + x$.

Exercise 8: Let the space of polynomials of degree at most 4 over $\mathbb{R}$ be equipped with basis $A = (x^4 + x^3,\ x^3 + x^2,\ x^2 + x,\ x + 1,\ x^4 + 1)$. Determine the matrix $[D_x]_{AK}$ for the mapping D_x that assigns $f(x)$ its derivative $f'(x)$.

(Consider $K = (x^0, \dots, x^4)$ as the canonical basis.)