Tutorial 4, October 24, 2019

1. Let $A \subset X \subset M$ be subsets in a metric space (M, d). Prove that A is compact in M if and only if it is compact in the subspace X.

2. Prove that two homeomorphic metric spaces are both compact or both non-compact.

3. (HW, 3 pts) How does the existence of a bijection between $[a, b]$ ($a < b$ are real numbers) and S_1 follow from the Cantor–Bernstein theorem?

4. Prove that the union of two connected and intersecting subsets in a metric space is a connected set.

5. Give an example of a connected union of two disconnected subsets in a metric space.

6. (HW, 4 pts) Is the Euclidean space $X \subset \mathbb{R}^2$, given by

   $$X = (\{0\} \times [-1, 1]) \cup \{(t, \sin(1/t) \mid 0 < t \leq 1)\} ,$$

   connected?

7. Prove that the Euclidean spaces $[0, 1] \times [0, 1] \subset \mathbb{R}^2$ and $S_1 \subset \mathbb{R}^2$ (the unit circle in the plane) are not homeomorphic.

8. Prove that the Euclidean spaces $\mathbb{R}^2$ and $S_2 \setminus \{(0, 0, 1)\} \subset \mathbb{R}^3$ (the unit sphere in $\mathbb{R}^3$ with the north pole deleted) are homeomorphic.

9. Is the Euclidean subspace $\mathbb{Q} \subset \mathbb{R}$ complete?

10. Is the intersection of two complete sets in a metric space complete?

11. (HW, 3 pts) And union?

12. And set difference?