Tutorial 3, October 17, 2019

1. State Heine's definition of continuity and prove that it is equivalent with the original definition.

2. Prove that both versions of topological definition of continuity, with open and with closed sets, are equivalent with the original definition.

3. Prove that every closed subset of a compact space is compact.

4. (HW, 3pts) Prove that every compact subset of a metric space is closed and bounded.

5. Recall the proof of the theorem: if $X \subset \mathbb{R}^n$ is closed and bounded set in an Euclidean space $\mathbb{R}^n$ then X is compact.

6. Prove that the image of a compact set by a continuous map is a compact set.

7. Recall the proof of the maximum-minimum principle.

8. Why can we in the proof of the Heine–Borel theorem restrict to the case of the whole space?

9. Why sequence $(a_n) \subset M$ satisfying $d(a_m, a_n) \geq \delta_0 > 0$ whenever $m \neq n$ does not have any convergent subsequence?

10. (HW, up to 5pts) Prove the characterization of open and closed sets in a subspace via open and closed sets of the whole space.

11. (HW, 3pts) Let $(M, d) = (\mathbb{R}^2, d_2)$ and $X \subset M$ be given by $X = \{(x, y) \in \mathbb{R}^2 \mid 0 < x^2 + y^2 < 1\} \cup \{(4, 4)\}$. Describe the inner, outer, border, limit and isolated points of the set X.

12. Prove that the Euclidean spaces (a, b), $a < b$ are real, and $\mathbb{R}$ are homeomorphic.