

Tutorial 1, October 3, 2019

1. Prove the triangle inequality for the Hamming metric on length n words.
2. Prove the triangle inequality for the graph metric.
3. (HW 3 pts.) Show that the axioms of metric space imply that metric is nonnegative.
4. Is symmetry implied by the other axioms of metric?
5. (HW 3 pts.) Prove the triangle inequality for the Euclidean plane distance.
6. Same in \mathbb{R}^n . Hint: use Cauchy–Schwarz—prove it as well.
7. (HW 6 pts.) Prove that no spherical cap (a part of sphere cut off by a plane) with the spherical metric is isometric to a set in \mathbb{R}^n with the Euclidean metric. (This is probably harder.)
8. Can any spherical triangle be isometrically realized in the plane (with Euclidean metric)?
9. Prove this counter-intuitive property of ultrametric spaces: every triangle is isosceles (two sides have equal lengths).
10. Check that the p -adic metric is really an ultrametric.