

## Tutorial 12, December 19, 2019

1. Expand  $f(x) = |x|$  on  $-\pi \leq x \leq \pi$  in Fourier series and determine its sum.
2. The same for  $f(x) = \cos(x)^3$  on  $\mathbb{R}$ .
3. The same for  $f(x) = \sin(x)^3$  on  $\mathbb{R}$ .

4.

$$\lim_{n \rightarrow \infty} \int_{-\pi}^{\pi} \frac{\sin(nx)}{\sin x} dx = ?$$

### Homeworks — due by January 2, 2020

1. (4 pts.) Expand  $f(x) = \exp(ax)$  on  $[-\pi, \pi)$ , where  $a \in \mathbb{R}$  is a parameter, in Fourier series.
2. (6 pts.) Let  $f(x) = a_0 + \sum_{k=1}^n (a_k \cos(kx) + b_k \sin(kx))$  be a trigonometric polynomial with coefficients  $a_k, b_k \in \mathbb{R}$ . Prove that if  $f(x) = 0$  for every  $x \in \mathbb{R}$  then all its coefficients are 0.
3. (3 pts.) Is there a Riemann-integrable function  $f : [-\pi, \pi) \rightarrow \mathbb{R}$  whose sine Fourier coefficients are  $b_n = 10/\sqrt{n}$ ?