# Tutorial 11, December 12, 2019

1. Give 4 examples of power series $\sum a_n x^n$ with conv. radius $R$, $0 < R < +\infty$, for each of the four possibilities of (non)convergence at $x = -R$ and $x = R$.

2. Yes or no: there exist two power series $\sum a_n x^n$ and $\sum b_n x^n$ differing in only finitely many coefficients, which have same conv. radii.

3. Yes or no: if $\sum_{n=1}^{\infty} f_n \rightrightarrows$ on $M$, then $\sum_{n=1}^{\infty} f_{2n} \rightrightarrows$ on $M$.

4. Yes or no: if $\sum_{n=1}^{\infty} f_n \rightrightarrows$ on $M$ by the Weierstrass test, then $\sum_{n=1}^{\infty} f_{2n} \rightrightarrows$ on $M$ by the Weierstrass test.

5. Compute the folowing limits.
   (a) $\lim_{x \to 1^-} \sum_{n \geq 1} (x^n - x^{n+1})$
   (b) $\lim_{x \to 1^-} \sum_{n \geq 1} \frac{(-1)^n}{n} \frac{x^n}{x^n + 1}$
   (c) $\lim_{x \to 0^+} \sum_{n \geq 1} \frac{1}{2^n n^x}$

6. (HW 3 pts.) Yes or no: If $f_n : \mathbb{R} \to \mathbb{R}$ is a sequence of functions such that $\sum_{n=1}^{\infty} \|f_n\|_{\infty} = +\infty$, then the series of functions $\sum_{n=1}^{\infty} f_n$ does not converge uniformly on $\mathbb{R}$. Justify your answer. (Reversal of the Weierstrass test.)

7. (HW 3 pts.) Find conv. radii of the following power series. Determine convergence at the endpoints of the convergence interval.
   (a) $\sum_{n \geq 1} x^n / n$
   (b) $\sum_{n \geq 0} 4^n x^{2n}$
   (c) $\sum_{n \geq 1} (-1)^{n+1} \frac{x^{2n}}{n 2^n}$

8. (HW 3 pts.) Prove: if $f(x) = \sum_{n \geq 0} a_n x^n$ is a power series with $R > 1$ and such that $f(1/k) = 0$ for every $k = 1, 2, \ldots$, then $a_n = 0$ for every $n \geq 0$.