Addendum

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to M. Klazar, Twelve countings with rooted plane trees, *Europ. J. Combinatorics*, **18** (1997), 195–210. It was brought to my attention by F. Ruskey and J. W. Moon that there is a considerable overlap of the results in the article with several other papers. I did not know of them when preparing my article and I apologize to their authors for my oversights.

The explicit formula for $F_1(x)$ (GF for antichains) was derived already by Ruskey [9]. Meir and Moon [4] treated Problem 1 (counting antichains) from a broader perspective, they derived the relation for $F_5(x)$ and the asymptotics of $w_5(n)$ (counting connected sets). The relation for $F_7(x)$ and the asymptotics of $w_7(n)$ (counting maximal independent sets) were found also by Meir and Moon [5]. Kirschenhofer et al. [2] solved Problem 6 (counting independent sets) and found the asymptotics, Theorem 4.2 is contained in their Corollary 2, p. 117.

I use this opportunity to add some more comments, references and corrections. In Table 1 the entry $w_1(4)$ should be 29. Problem 6 is investigated for other tree families in Kirschenhofer et al. [1]. The inductive argument of Theorem 5.1 is generalized in Prodinger [6]. A reference to Theorem 5.1 earlier than those given is Prodinger and Urbanek [8]. The earliest reference to Theorem 5.2 seems to be p. 70 in Knuth [3] (Exercise 20). In remark 6.3 on p. 209 the values of $M_8(n), m_9(n)$, and $M_6(n)$ should be $2^{n-1}+n-1, n$, and $2^{n-1}+1$, respectively. The value $m_{10}(n)=n-1$ is erroneous, $m_{10}(n)$ can be smaller.

As to the problems stated on p. 210, Prodinger and Tichy [7] proved that $m_6(n)=F_{n+1}$ ($F_0=F_1=1, F_n=F_{n-1}+F_{n-2}$). In a similar way one can show that $M_9(n)=F_n$. Wilf [10] proved that $M_7(n)$ is $2^{n/2-1}+1$ for n even and $2^{(n-1)/2}$ for n odd. In the lower bound for $M_{12}(n)$ delete the 4 and replace $1/n$ by n. As to the very last problems on p. 210, J. Fiala from Liberec constructed infinitely many trees with the same chain and antichain number. The author proved that there are numbers with arbitrary large height.

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