EXERCISES FOR TUTORIAL 9 OF MA 2, November 27, 2024

Review of the Riemann integral of real functions of one (real) variable.

1. Using your favorite definition of the Riemann integral compute $\int_0^1 x \,\mathrm{d}x$.
2. Using your favorite definition of the Riemann integral prove that if the function $f\colon [a,b] \to \mathbb{R}$ is unbounded then the integral $\int_a^b f$ does not exist.
3. Using your favorite definition of the Riemann integral prove that if the integral $I := \int_a^b f$ exists (and $a \le b$), then we have the inequality

$$|I| \le (b-a) \cdot \sup(\{|f(x)| \mid a \le x \le b\}) \,.$$

4. We have seen this already but still: give examples of Riemann integrable and nonnegative functions $f\colon [a,b] \to [0,+\infty)$ such that $\int_a^b f = 0$, but $f \neq 0$ (f is not constantly 0).
5. Give an example of a continuous and bounded function $f\colon (0,1] \to \mathbb{R}$ that is not uniformly continuous.