EXERCISES FOR TUTORIAL 7 OF MA 2, Nov 13, 2024

Two more HWs on compact sets, where $M = (M, d)$ is a metric space. Then three problems on determinants, where $M \in \mathbb{Z}^{n\times n}$ is a square $n \times n$ matrix with integer entries.

1. Let $A, B \subset M$ be compact sets. Prove by the sequential definition of compactness that also $A \cup B$ is compact.
2. Prove the same via the definition of compactness by open covers.
3. Show that $\det M$ is an integer.
4. Let $M = (a_{i,j})_{i,j=1}^{n}$ and $|a_{i,j}| \le A$ for every $i, j = 1, 2, \ldots, n$ ($A \ge 0$ is a real number). Then $|\det M| \le ?$. That is, estimate the determinant from above in terms of A and n.
5. Let M be regular and such that its inverse M^{-1} is (like M) integral, i.e., $M^{-1} \in \mathbb{Z}^{n\times n}$. Show that then $\det M = \pm 1$.