EXERCISES FOR TUTORIAL 10 OF MA 2, Dec 4, 2024

More review of the Riemann integral, but also of multivariate functions. For real numbers $a < b$ we denote by $\mathcal{R}(a,b)$ the set of Riemann-integrable functions $f\colon [a,b] \to \mathbb{R}$.

1. Let the function $f\colon [0,1] \to \mathbb{R}$ be given by $f(1/n) := 1$ for $n = 1, 2, \ldots$ and by $f(x) := 0$ otherwise. Is $f \in \mathcal{R}(0,1)$? If so, compute $\int_0^1 f$.
2. Prove the implication $f, g \in \mathcal{R}(a,b) \Rightarrow f + g \in \mathcal{R}(a,b)$.
3. Prove the implication $f, g \in \mathcal{R}(a,b) \Rightarrow fg \in \mathcal{R}(a,b)$. In this and the previous problem you do not have to use definitions of the R. integral, and instead you can refer to a theorem on the R. integral.
4. Define the Riemann integral of functions with several variables.
5. Define uniform continuity of a map $f\colon M \to N$ between two metric spaces (M,d) and (N,e).