EXERCISES FOR MA 2 TUTORIAL 1, Oct 2, 2024

First **conditions for getting credits for the tutorial**. You need to work out $\geq \frac{3}{5}$ of the exercises and get $\geq \frac{1}{2}$ of the points in the test given on the last tutorial in December. For every tutorial, except the last two, a set of five exercises will be posted here before the tutorial. Please, send me your solutions via e-mail (in legible form) to `klazar@kam.mff.cuni.cz` at the latest by the next Monday/Tuesday midnight after the tutorial. I will discuss solutions on the coming Wednesday tutorial, and bring your solutions with my comments. Please try to be concise and BRIEF in your solutions, there are cca 40 of you in the tutorial and I have neither patience nor time for reading (and correcting) long memoirs.

If I see an honest effort in your solution to solve the exercise, I count it as correct no matter what.

1. Write some set-theoretic definition of a function $f\colon A \to B$. What is the definition domain and the range of f? For sets X and Y, define the sets $f[X]$ and $f^{-1}[Y]$.

2. Write the definition (axioms) of a metric space $(X,\, d)$. Show that non-negativity of the metric d follows from other axioms.

3. For real $a < b$ we denote by $\mathcal{R}(a, b)$ the set of functions $f\colon [a, b] \to \mathbb{R}$ that have Riemann integral on $[a, b]$. For $f, g \in \mathcal{R}(a, b)$ we define

$$d(f, g) \equiv \int_a^b |f(x) - g(x)| \,\mathrm{dx}\ .$$

   Is $(\mathcal{R}(a, b), d)$ a metric space?

4. Define open sets, and balls $B(x, r)$ (called $\Omega(x, r)$ in the lecture) with the center $x \in X$ and radius $r > 0$ in a metric space (X, d). Show that every ball is an open set.

5. For sets A and B define their Cartesian product $A \times B$. Prove: if A and B are nonempty then

$$A \neq B \Rightarrow A \times B \neq B \times A\ .$$