EXERCISES FOR TUTORIAL 7 OF MA 2, Nov 23, 2023

The first three problems concerning Taylor expansions of functions of several variables and free extremes are to be solved using the theory explained in
`https://kam.mff.cuni.cz/%7Eklazar/pr7MAIII07.pdf`
and
`https://kam.mff.cuni.cz/%7Eklazar/pr9MAIII07.pdf`
(in Czech)
or
`https://kam.mff.cuni.cz/%7Eklazar/pr7MAIII07eng.pdf`
and
`https://kam.mff.cuni.cz/%7Eklazar/pr9MAIII07eng.pdf`
(brief text in English)

1. Let $f(x, y, z) = x^3 + 2y^2 - 2xz + 3$. Compute the complete Taylor expansion of this functions with center in $(0, 0, 0)$. Explain the result.

2. Let $f(x, y) = \sin x + \cos y$. Compute the complete Taylor expansion of this functions with center in $(\frac{\pi}{2}, \frac{\pi}{2})$.

3. Let again $f(x, y) = \sin x + \cos y \colon \mathbb{R}^2 \to \mathbb{R}$. Using partial derivatives find (local and global) extremes of this functions.

4. Using Lagrange multipliers find (local and global) extremes of the function $f(x, y) = x + y$ on the set $M = \{(x, y) \mid x^2 - 2x + y^2 + 1 = 2\}$. Explain your solution geometrically.

5. The same for the function $f(x, y) = x^2 + y^2$ on the set M that equals to the circle with center in $(10, 10)$ and radius 2.