EXERCISES FOR TUTORIAL 4 OF MA 2, Oct 26, 2023

1. Suppose that the map $f = (f_1, f_2, f_3)\colon \mathbb{R}^2 \to \mathbb{R}^3$ is given by $f_i(x, y) = x^i + y^i$, $i = 1, 2, 3$. Compute the matrix $D\,\mathbf{f}$ of the total differential (in a general point $(x, y) \in \mathbb{R}^2$).
2. Which of the intervals $I = [0, 1)$, $[0, 1]$ and $[0, +\infty)$ has the property that every sequence $(a_n) \subset I$ has a convergent subsequence with the limit in I? Justify your answer.
3. Prove that every finite metric space is compact.
4. Let $(X, |x - y|)$, $X = \{0\} \cup \{1/n \mid n = 1, 2, \ldots\} \subset \mathbb{R}$, be an Euclidean subspace of the real axis. Is it compact? Justify your answer.
5. Is the intersection of two compact subsets of a metric space always a compact set? Justify your answer.