EXERCISES FOR TUTORIAL 3 OF MA 2, Oct 19, 2023

Recall that in an *ultrametric space* $X = (X, d)$ (briefly UMS), which is a special kind of a metric space, the *strong triangle inequality* holds: $d(x, y) \le \max(\{d(x, z), d(z, y)\})$.

1. Show that in any UMS in any triangle some two sides have equal lengths — every triangle is isosceles.
2. Prove that in any UMS in any ball $B(a, r)$ any point $b \in B(a, r)$ can serve as the center (of $B(a, r)$).
3. In MA 2 one has to know some linear algebra. Let $k, l, m, n \in \mathbb{N}$ (these are four natural numbers) and $A \in \mathbb{R}^{k\times l}$, $B \in \mathbb{R}^{l\times m}$ and $C \in \mathbb{R}^{m\times n}$ are three real matrices with the stated dimensions. Define the matrix product $A \cdot B$.
4. Prove that it is associative:
$$(A \cdot B) \cdot C = A \cdot (B \cdot C)\ .$$
5. Let $f\colon \mathbb{R}^3 \to \mathbb{R}$ and $g_i\colon \mathbb{R}^2 \to \mathbb{R}$, $i = 1, 2, 3$, are maps given by the formulas $f(x, y, z) = x^2 + 2y^2 + 3z^2$, $g_1(x, y) = 2x + y$, $g_2(x, y) = \sin(x + y)$ and $g_3(x, y) = \cos(x + y)$ and let
$$h(x,\, y) = f(g_1(x,\, y),\, g_2(x,\, y),\, g_3(x,\, y))$$
be the composite map. Use the chain rule to compute the partial derivatives
$$\frac{\partial h}{\partial x} \text{ and } \frac{\partial h}{\partial y}\ .$$