EXERCISES FOR MA 2 TUTORIAL 2, Oct 12, 2023

1. Give an example of a metric space (X, d) and a sequence $A_n \subset X$, $n = 1, 2, \ldots$, of open sets in it such that the set $\bigcap_{n=1}^{\infty} A_n$ is not open.
2. Give an example of a metric space (X, d) and a sequence $A_n \subset X$, $n = 1, 2, \ldots$, of closed sets in it such that the set $\bigcup_{n=1}^{\infty} A_n$ is not closed.
3. Let $f\colon X \to Y$ be a map between metric spaces (X, d) and (Y, e) and let $a \in X$. Define what it means that f is continuous in a.
4. Let $f\colon X_1 \to X_2$ and $g\colon X_2 \to X_3$ be maps between metric spaces (X_i, d_i), $i = 1, 2, 3$, $a \in X_1$, $b := f(a) \in X_2$, f be continuous in a and g be continuous in b. Show that the composite map $g \circ f = g(f)\colon X_1 \to X_3$ is continuous in a.
5. Let $f\colon \mathbb{R}^2 \to \mathbb{R}$ be the map given in the lectures, i.e. $f(x, y) = xy/(x^2 + y^2)$ for $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$ ($\mathbb{R}^2$ and $\mathbb{R}$ are endowed with euclidean metrics). Find $(\partial f/\partial x)(0, 0)$ and $(\partial f/\partial y)(0, 0)$ and show that f is not continuous in the point $(0, 0)$.