EXERCISES FOR MA 2 TUTORIAL 1, Oct 5, 2023

First **conditions for getting credits for the tutorial**. You need work out $\geq \frac{3}{5}$ of the exercises and get in the test on the last tutorial $\geq \frac{1}{2}$ points. For every tutorial, except the last one, a set of five exercises will be posted here before the tutorial. Please, send me your solutions via e-mail (in legible form) to `klazar@kam.mff.cuni.cz` at the last by the next Tuesday/Wednesday midnight after the tutorial. I will discuss solutions on the Thursday tutorial (and may bring to some of you your solutions with my comments).

1. Write some set-theoretical definition of a function $f\colon A \to B$. What is the definition domain and the range of a function? For $A' \subset A$ and $B' \subset B$, define the sets $f[A']$ and $f^{-1}[B']$.

2. Write the definition (axioms) of a metric space

$$(X,\, d)\ .$$

   Show that nonnegativity of the metric d follows from other axioms.

3. For real $a < b$ we denote by $\mathcal{R}(a,b)$ the set of functions $f\colon [a,b] \to \mathbb{R}$ that have Riemann integral on $[a,b]$. For $f, g \in \mathcal{R}(a,b)$ we define

$$d(f,g) := \int_a^b |f(x) - g(x)|\,\mathrm{dx}\ .$$

   Is $(\mathcal{R}(a,b), d)$ a metric space?

4. Define open sets, and balls $B(x,r)$ (called $\Omega(x,r)$ in the lecture) with the center $x \in X$ and radius $r > 0$ in a metric space (X,d). Show that every ball is an open set.

5. For sets A and B define their Cartesian product $A \times B$. Prove: if A and B are nonempty then

$$A \neq B \Rightarrow A \times B \neq B \times A\ .$$