
Problem 1. Give an example of a partial order on X where X has 15 elements such that width of your partial order is 3 and length is 5.

Problem 2. Let X be a finite set.

1. Prove that $(2^X, \subseteq)$ is a poset.
2. Draw the Hasse diagram for this poset when X is $\emptyset, [1], [2], [3]$

Problem 3. Let $p \in \mathbb{N}, p \geq 2$. Define the relation R over $\mathbb{N}$ as follows: xRy iff p divides $x - y$.

1. Prove that R is an equivalence relation.
2. Identify the equivalence classes of R .

Problem 4. Prove that a minimum element, if it exists, is unique.

Problem 5. Prove that a minimum element is also a minimal element.

Problem 6. Prove or disprove: If a poset has a single minimal element, then it is also the minimum element.

Problem 7. Prove by mathematical induction that a finite poset always has a maximal element.

Problem 8. Let R be a relation over a set X , and let $Y \subseteq X$.

1. Prove that if R is an ordering over X then $R \cap (Y \times Y)$ is an ordering over Y .
2. Prove that if R is an equivalence relation over X then $R \cap (Y \times Y)$ is an equivalence relation over Y .

Problem 9. Let R and S be arbitrary equivalence relations (resp. orderings) on the set X . Determine which of the following are necessarily equivalence (resp. ordering):

1. $R \cap S$
2. $R \cup S$
3. $R \setminus S$
4. $R \circ S$

Problem 10. Prove that every partial ordering on a finite set X has at least one linear extension. For a partial ordering $\leq$, linear extension is any linear order $\preceq$ such that $x \preceq y \implies x \leq y$.